Lateral Resolution of Apodised Two-Annuli Coded-Aperture Imaging Systems

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Abstract: The Point Spread Function (PSF) of two-annuli coded- aperture imaging systems has been investigated by a rigorous Fourier analytical method. The full-width at Half-Maxima (FWHM) obtained from it has been used as a measure of the spatial resolution of the system. The effects of defocusing and apodising the aperture have been considered. The variation of the FWHM as a function of the location of the first annulus has been studied

IndexTerms: **Coded – Aperture, Resolution and Apodisation.**

I. INTRODUCTION

Various forms of coded apertures have been proposed for use in nuclear medicine, astronomy, controlled fusion studies and nuclear safety research (Barret and Swindell, 1981). In multiplexed radiographic imaging systems, a few dilute coded apertures, like a single annulus, two-annuli, ring delta, etc., have been introduced (Simpson, 1978). However, a single annulus coded aperture suffers from a particular drawback because of a specific disadvantage known as the 'Glitch Artifact" which puts a limit on the size of the object field.

It has been shown that this so called glitch artifact can be eliminated by a suitable choice of combinations of annular width and mean radii of two-annuli coded aperture imaging systems. This is because of the fact that the various zeros in the transforms of the individual annuli will occur at different locations. Hence, two-annulus coded apertures have some advantage over a single annulus imaging systems. Now, in multiplexed radiographic imaging systems with high energy x-rays or gamma rays, a precise knowledge of the spatial resolution of these systems is very important. As the usual measure of spatial resolution by choosing point or line pairs as test targets is qualitative and subjective, a Fourier analytical treatment for a more rigorous and complete description of spatial resolution based on the considerations of the point spread function (PSF) of the system has been suggested (Mertz and Dol, 1979).

Following this idea, we have utilized the principles of Fourier transform optics to evaluate the PSF of two-annuli coded aperture imaging systems. We have then studied the lateral resolution of the system on the basis of the full width at half maximum (FWHM) point (Shung, Smith and Tsui, 1992). The effects of using amplitude filters with "apodisation", a widely used current technique of modifying the PSF of the imaging system through a suitable manipulation of the aperture transmission have been investigated.

2. THEORY:

The amplitude PSF of a single annulus coded aperture imaging system can be written as (Boivin, 1964) as

$$A(y,z) = 2\int_{\varepsilon}^{\rho} f(r)\, J_0\,(z\,r)\, e^{-i\,y\,r^2/2}\; r\; dr \tag{1}$$

Where and are, respectively, the inner and outer radii of the aperture with. Whereas ε and ρ are the dimensionless diffraction variables along and perpendicular to the axis of the system; y = 0 corresponds to the perfectly focused plane of observation; represents the amplitude transmission function of the apodiser used; being the reduced fractional co-ordinate of an arbitrary point on the aperture whose radius has been normalized to unity. Is the Bessel function of the first kind and order zero. For a two-annuli coded aperture system (Fig.1) the above expression will assume the following form.

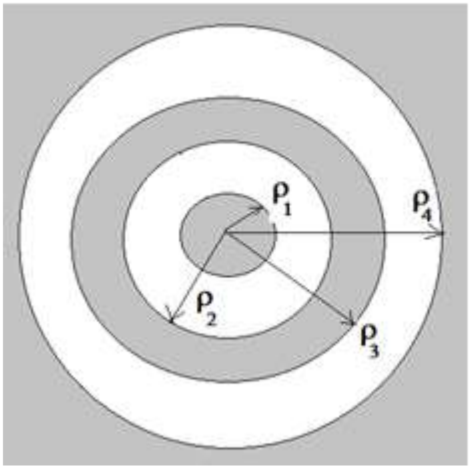

Fig. 1 Two-Annuli Coded Aperture

$$A(y,z) = 2\int_{\rho_1}^{\rho_2} f(r)\, J_0(z r)\, e^{-i y r^2/2}\, r\, dr$$

$$A(y,z) = 2\int_{\rho_3}^{\rho_4} f(r)\, J_0(z r)\, e^{-i y r^2/2}\, r\, dr \qquad (2)$$

In the above ρ_1 and ρ_2 represents the inner and the outer radii of the first annulus of width $\rho_2 - \rho_1$ and the ρ_3 and ρ_4 are the inner and the outer radii of the second annulus having the width $\rho_4 - \rho_3$. The four radii by the limits. $0 \le \rho_1 \le \rho_2 \le \rho_3 \le \rho_4 \le 1$

Now, the intensity PSF can be obtained by taking the square modulus of the amplitude PSF. Thus,

$$B(y, z) = |A(y, z)|^2 \qquad (3)$$

For studying the effects of apodisation, we have considered an amplitude filter whose pupil function can be represented by

$$f(r) = \beta r^2 \qquad (4)$$

Where β is known as the apodisation parameter which normally varies from 0 to 1. It is this parameter which controls the point-to point non-uniform transmission of the aperture. In the particular case of $\beta = 0$ corresponds to the diffraction limited Airy systems with uniform transmission of the pupil.

3. RESULTS AND DISCUSSION:

Equation (2) and (4) has been used to evaluate the intensity PSF of two-annuli coded aperture imaging systems by utilizing Matlab 7.2. In the present study, the widths of the two annuli equal and constant through $\rho_2 - \rho_1 = \rho_4 - \rho_3 = 0.1$.

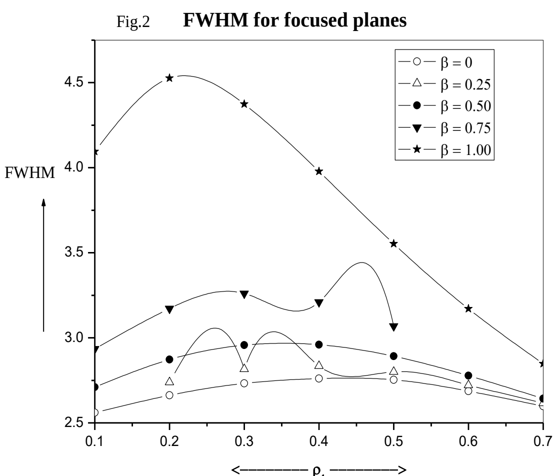

Fig 2 FWHM for Focused Planes

Further, the second annulus has been kept fixed at the outer periphery of the aperture while the position of the first annulus has been varied to investigate the effects of its location on the pupil. The FWHM has been estimated by using its definition as twice the value of 'z' for which B of (0,Z) is 0.5 times of the peak intensity (Shung, Smith and Tsui,1992).

TABLE - 1 FWHM VALUES

y	ρ1	β = 0	β = 0.25	β = 0.5	β = 0.75	β = 1.00
	0.10	2.5598	2.6136.	2.7102	2.9344	4.0948
	0.20	2.6628	2.7388	2.8724	3.1726	4.5262
	0.30	2.7326	2.8152	2.9574	3.2604	4.3734
0	0.40	2.7604	2.8346	2.9590	3.2098	3.9778
	0.50	2.7536	2.7996	2.8914	3.0684	3.5530
	0.60	2.6868	2.7216	2.7782	2.8860	3.1708
	0.70	2.5986	2.6154	2.6430	3.6968	2.8474
	0.10	2.6218	2.6910	2.8144	3.0978	4.5260
	0.20	2.7532	2.8492	3.0168	3.3856	4.9492
	0.30	2.8362	2.9378	3.1096	3.4648	4.6418
π/2	0.40	2.8600	2.9476	3.0910	3.3686	4.1290
	0.50	2.8244	2.8874	2.9880	3.1744	3.6354
	0.60	2.7414	2.7786	2.8382	2.9474	3.2134
	0.70	2.6274	2.6446	2.6726	2.7260	2.8670
	0.10	3.5752	3.8536	4.3288	5.3688	11.8388
	0.20	3.9978	4.3058	4.7894	5.6566	7.4306
	0.30	4.0336	4.2548	4.5676	5.0372	5.7842
π	0.40	3.8044	3.9316	4.1020	4.3440	4.7044
	0.50	3.4754	3.5410	3.6292	3.7536	3.9824
	0.60	3.1402	3.1712	3.2140	3.2778	3.3826
	0.70	2.8376	2.8502	2.8690	2.8996	2.9590
	0.10	1.9562	1.8290	1.5704	-----	-------
	0.20	1.6762	1.4352	0.8222	13.4636	11.1424
	0.30	1.3916	0.9784	------	10.8466	8.1554
3π/2	0.40	1.1500	------	-------	9.0012	6.0546
	0.50	1.1324	------	9.4292	7.6704	4.7586
	0.60	1.5472	1.2738	--------	7.5296	4.0272
	0.70	2.1238	2.0868	2.0092	1.7448	4.5384
	0.10	2.1484	2.0844	1.9624	1.6328	-------
	0.20	2.0146	1.9092	1.6988	1.0194	12.3026
	0.30	1.9152	1.7796	1.4932	------	9.5926
2π	0.40	1.8952	1.7598	1.4642	20.9194	7.6156
	0.50	2.0084	1.9204	1.7348	1.0424	6.6366
	0.60	2.2372	2.2150	2.1722	2.0540	------
	0.70	2.4278	2.4348	2.4468	2.4746	2.5986

The variations of FWHM with ρ1, the inner radius of the first annulus have been shown in Fig.2. Curves have been drawn for different values of β varying from 0 to 1. These results are for the focused planes of observation corresponding to y = 0. In all these curves, the FWHM is found to increase initially with ρ1 to reach a maximum and then decrease for higher values of ρ1 . Also, for higher values of β, viz. β=1, the increase is more rapid and the fall is more steep. Further, the values of ρ1 at which the FWHM becomes maximum are found to decrease with β. For example, for β = 0, the FWHM is maximum for ρ1 = 0.4 and for β = 1, it is maximum for ρ1 = 0.2.

It is well known that a single narrow annulus reduces the width of the central lobe while increasing the energy in the side lobes of the intensity PSF. In the present case, the second annulus is at the periphery of the aperture and is fixed there with a constant width of ρ4-ρ3= 0.1. This will increase the contributions of the high frequency components in the image intensity distributions with the reduction in the width of the resultant PSF. The contributions from the first annulus and its gradual shifting towards the second adds up to this effect of narrowing down of the main lobe of the PSF. As a consequence, the value of the FWHM decreases though there is an initial increase for lower values of ρ1 . The effects of defocusing of the FWHM have been shown in Fig3 for various amounts of defocusing, with the inner annulus shifting from the center to the periphery towards the second annulus. For smaller amounts of defocusing, viz., y=π/2 and π and also in the focused plane (y=0), the FWHM initially increases and then decreases, finally. However, for higher values of defocusing, viz., y=3π/2 and 2 π, a comparatively opposite trend takes place. The FWHM values first decrease attains least value and then increase, finally. Another important point to be mentioned is the appearance of non-zero minima in the PSF in the presence of defocusing.

Some of these non-zero minima have intensity values not at all negligible compared to that of the first side-lobe in the PSF in the absence of defocusing. This observation justifies more in selecting the FWHM as a better criterion for the measure of lateral resolution of a coded aperture imaging system. Table-1 lists the FWHM values for various combinations of y, ρ1, and β with $\rho_3 = 0.9$ and $\rho_4 = 1$ through ρ_1 is made to increase from 0.1 to 0.7 with $\rho_2 = \rho_1 + 0.1$.

Fig.3 FWHM for Defocused planes

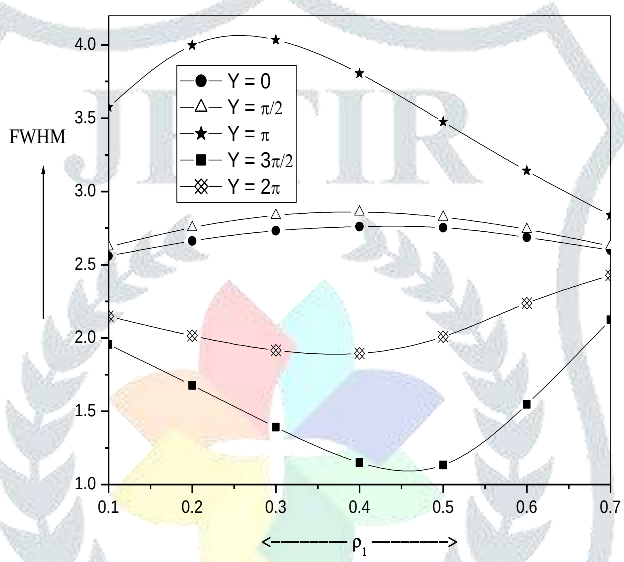

Fig 3 FWHM for Focused Planes

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