Formulation of the forward kinematic model of two-degree of freedom RP planar manipulator

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DENAVIT - HARTENBERG (D-H) CONVENTION

A commonly used convention for selecting frames of reference in robotic application is Denavit-Hartenberg convention. In this convention, the position and orientation of the end-effector is given by

$$H = {}^{n}T_0 = A_1 A_2 \ldots A_n$$

where,

$$A_i = \begin{bmatrix} {}^{i}R_{i-1} & {}^{i}d_{i-1} \\ 0 & 1 \end{bmatrix}$$

The homogeneous transformation A_i is represented as a product of four basic transformations as follows:

$$A_i = Rot(z_i, \theta_i) Trans(z_i, d_i) Trans(x_i, a_i) Rot(x_i, \alpha_i)$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -c\alpha_i s\theta_i & s\alpha_i s\theta_i & a_i c\theta \\ s\theta_i & c\alpha_i c\theta_i & -s\alpha_i c\theta_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Formulate the forward kinematic model of two-degree of freedom RP planar manipulator as shown in figure 3.9. Use classical D-H convention.

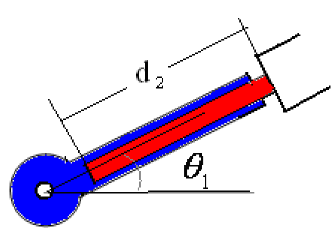

Figure 3.9: Two-degree of freedom RP planar manipulator

Solution: The manipulator consists of two joints. The first joint is revolute joint. The second joint is prismatic joint. The frame assignment of the manipulator is shown in figure 3.10. For revolute joint, $d = 0$ and for prismatic joint, $\theta = 0$. The axis of revolute joint is perpendicular to the paper and the axis of prismatic joint is within the plane of paper. The axis of revolute joint and the axis of prismatic joint intersect each other.

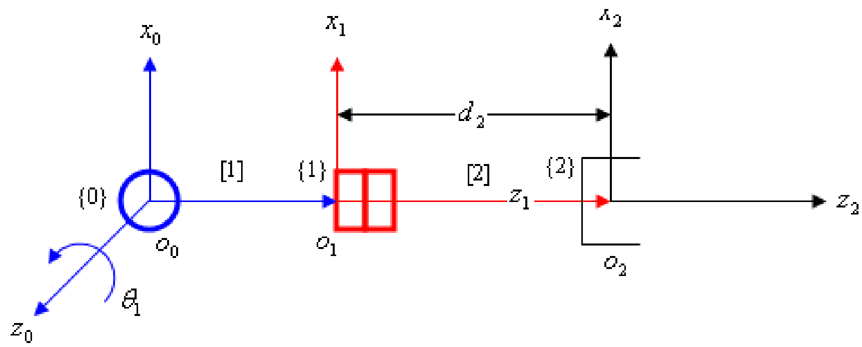

Figure 3.10: Frame assignment for two-degrees of freedom RP planar manipulator

Step -1**:** The joints are revolute and prismatic. The two links are numbered [1] and [2]. The base frame is {0} and frames for the rest of links are numbered {1} and {2}.

Step -2: The location of frame to the base is arbitrary. The x_0 axis, which is perpendicular to z_0, is chosen to be parallel to x_1 when the first joint angle variable θ_1 = 0 in the home position. The y_0 axis is defined by the cross product $y_0 = z_0 \times x_0$.

For i = 1, perform Steps 3 to 5.

Step -3: The link 1 has two joint axes, z_0 and z_1. The z_0 axis is assigned to joint 1. The z_1 axis is assigned to joint 2. There is a common normal between z_0 and z_1 axes. Axis z_1 intersects z_0 axis. 0_1 is located at this point of intersection.

Step -4: Axis x_1 is established along the common normal between z_0 and z_1 through 0_1.

Step -5: The y_1 axis is defined by the cross product of $y_1 = z_1 \times x_1$.

. For i = 2

Step -3: The link 2 has one joint axis, z_1 which is common to link 1 and link 2. The second end of link 2 is rigidly connected to the end-effector. The z_2 is the axis of approach for the end-effector. Since z_1 and z_2 zxes are parallel and collinear, the origin o_2 is located in any convenient position along z_2 axis as shown in figure 3.10.

Step -4: The common normal between z_1 and z_2 axes is x_2. Since the joint 2 is prismatic, x_2 axis is chosen in the direction parallel to x_1 axis and passing through the origin o_2.

Step -5: The y_2 axis is defined by the cross product of $y_2 = z_2 \times x_2$.

Step -6: Establish the end-effector frame {2} as shown in figure 3.10.

Step – 7: The joint-link parameters are tabulated as follows:

Link, i	a_i	α_i	d_i	θ_i
1	0	90°	0	θ_1
2	0	0	d_2	0

Step -8: Form the homogeneous transformation matrices ${}^{i-1}T_i$ by substituting the above parameters into equation (3.6)

$$ {}^0T_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^1T_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Step- 9: Form ${}^0T_2 = {}^0T_1\,{}^1T_2$. This gives the position and orientation of the tool frame expressed in base coordinates.

$$ {}^0T_2 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & s_1 & d_2 s_1 \\ s_1 & 0 & -c_1 & -d_2 c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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