Regression Modelling Approach for the Analysis of Investment Shell Moulds from Coal Flyash

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ABSTRACT
Coating slurries for investment shells in the lost-wax process are refractory filler materials dispersed in a liquid binder. A variety of refractories are used., each one being unique in physical properties such as density and particle shape as well as having unique chemical properties. This article discusses the utilization of coal flyash as a refractory filler material. A factorial 23 experimental design was applied for the analysis of the properties of investment shell moulds.

INTRODUCTION

In the manufacture of investment shell moulds by the lost-wax process1, a multi-layered investment shell is built-up by repeatedly dipping a wax pattern into a slurry, draining and sprinkling with a coarse sand. Each individual coat is air-dried prior to applying the next coat. On achievement of the required thickness of the shell, the wax pattern is removed from the set-up, the shell is fired, and the mould is poured.

The materials used to build the investment shell, specially binders and refractories, play a vital role in the production of quality castings. The refractory materials include fused silica, alumina and zircon powder (2,6). Investment shells made from coating slurries containing zircon powder have a lower strength than shells made from silica flour and alumina. This is in relation to their densities: Zircon powder (4.6 gm/cc); Alumina (4.0 gm/cc) and Fused silica (2.6 g/cc). Refractory materials with low densities insure a more constant strength of coats and shells. However, the utility of fused silica is limited by its metal-mould reaction and by the high abrupt expansion at 573oC accompanying its α to β phase transition. As a result, shells containing fused silica must be fired slowly, a practice most industries find inconvenient. The main disadvantage of alumina is its poor resistance to thermal shock.

The refractory materials used in the preparation of coating slurries range from *200 to 500* mesh in particle size. Materials finer than this have surface energy, which results in sintering at appreciable lower temperature.

Coal Flyash is the residue of coal combustion. When coal is totally burnt, the non-combustible constituents of coal, particularly the oxides of silica and alumina convert into fine ash. About 80% of the ash files along with the flue gases and gets entrapped within bag filters or electrostatic precipitators and is identified as *flyash*. The estimation of flyash generation in India is shown in *Fig.1*

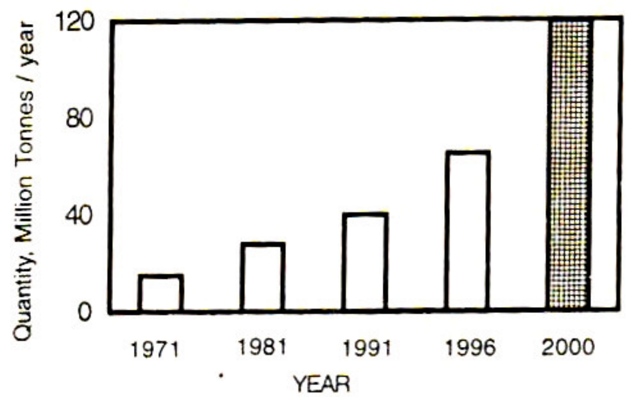

Fig. 1: Estimation of Coal Flyash Generation in India.

The main objective of the present work was adoption of regression modeling approach to analyse the properties of Investment Shell Moulds using coal flyash as refractory material. A factorial 2^3 matrix was used in the design of experiments.

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EXPERIMENTAL PROCEDURE

Materials Used

Binder	Silox-30 (Colloidal Silica Binder)
Filler	Coal Flyash (200 and 325 mesh)
Stucco Sand	River Sand (AFS Fineness Number 120 and 50)

Slurry was prepared by adding the refractory filler (i.e. coal-flyash) to the silox binder, using sufficient agitation to break-up agglomerates and thoroughly wet and disperse the coal-flyash. Dipping, draining and stuccoing of *bending* and *permeability specimens* were carried out manually. Six coats were given on the specimens. The shell making process is shown in *Fig.2*. The *bending* and *permeability specimens* were fired at 800oC for one hour. The bending test was conducted on an universal sand testing machine, and permeability meter was used to test permeability of the specimens [9-12].

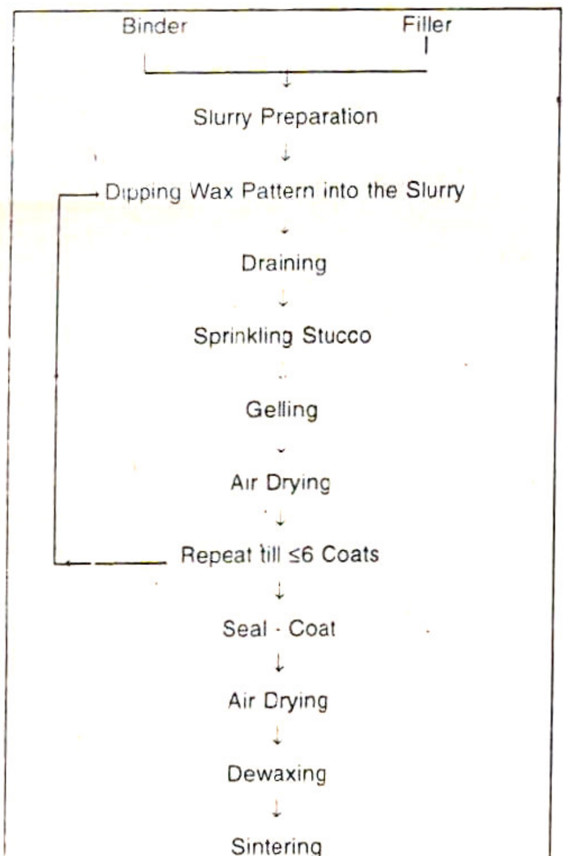

Fig. 2 : Shell Making Process

PROCESS MODELLING

Modelling such a complex system as investment casting process is not easy, and to facilitate the limited objective of analysis of shell moulds, the number of control factors and their levels should be kept to a bare minimum. The control factors selected are: *Grain Size of Refractory Filler, Filler to Binder (F/B) Ratio and Grain Size of Stuccoing Sand.*

A complete 2^3 factorial experimental model was formulated. The regression was affected assuming the optimization parameter, Y_i, is a random population normally distributed and the variance of Y_i does not depend upon its absolute value. The selected control factors and their code levels are given *Table -1*. The signs for contrasts in a 2^3 factorial experiment are given in *Table – 2*. For increased precision, each experiment was replicated twice.

The variance of optimization and the homogeneity of variance for *fired bending strength* and *hot permeability* were computed according to the standard procedures (*Appendix-1*).

Table - 1
Levels of Selected Control Factors

Level	Grain Size of Refractory Filler (X_1)	Filler to Binder Ratio *(g/cc)* (X_2)	Grain Fineness of Stucco Sand *(AFS GFN)* (X_3)
Upper (+1)	200 mesh	2.0	50
Lower (-1)	325 mesh	1.0	120

Table - 2
Signs for Contrasts in a 2^3 Factorial Experiment

Treatment Combination	X_1	X_2	X_3	X_1X_2	X_1X_3	X_2X_3	$X_1X_2X_3$
(1)	−	−	−	+	+	+	−
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
c	−	−	+	+	−	−	+
ab	+	+	−	+	−	−	−
ac	+	−	+	−	+	−	−
bc	−	+	+	−	−	+	−
abc	+	+	+	+	+	+	+

Mathematical Model of the Process

Linear and No-linear Models were chosen to study the investment casting process.

The Linear model is

$$Y = b_o + b_1X_1 + b_2X_2 + b_3X_3$$ and

The third order Non-linear Model is

$$Y = b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{23}X_2X_3b_{123}X_1X_2X_3$$

Where

X_1, X_2, X_3 = selected control factors

Y = optimization parameter.

The coefficient b_0 represents the response at the centre of experiment, and the coefficients b1, b2, b3, b12, b13, b23, b123 represent the *"Linear"* , *"Linear x Linear"*, *"Linear x Linear x Linear"* interaction effects of the factors X_1, X_2, and X_3 respectively.

The coefficients of regression were calculated using the *least-square method*. The adequacy of *linear* and *non-linear* models was confirmed by Fisher's Ratio (*Appendix-2*). The confidence intervals for regression coefficients were also computed (*Appendix-3*) at 5% significance level.

RESULTS AND DISCUSSION

The variance of optimization for *fired bending strength* and *hot permeability* were found to be 0.075 and 0.00815 respectively. The experimental values of *Cochran's Ratio* are 0.3267 and 0.0982 respectively for *fired bending strength* and *hot permeability*. The two values are less than 0.6798, the tabulated value for 2^3 matrix with two trials of each treatment. Therefore, the necessary condition for the application of regression analysis satisfied.

The *linear* regression equations are:

$$\textit{Fire Bending Strength} = 16.18 - 3.85X_1 + 7.68X_2 - 2.16X_3$$

$$\textit{Hot Permeability} = 12.03 + 4.25X_1 - 2.43X_2 + 1.38X_3$$

The experimental values of *Fisher's ratio* for *fired bending strength* and *hot permeability* are 80.58 and 734.97 respectively. The two values are greater than 6.09, which is the tabulated value of *Fisher's ratio* at 5% significance level for 4 degrees of freedom. Hence, a *linear model* is inadequate in the above two cases.

The non-linear regression equations with interaction of independent variables are:

$$\textit{Fired bending Strength} = 16.18 - 3.85X_1 + 7.68X_2 - 2.16X_3 - 0.75X_1X_2 - 0.09X_1X_3 - 0.99X_2X_3 + 0.01X_1X_2X_3$$

$$\textit{Hot Permeability} = 12.03 + 4.25X_1 - 2.43X_2 + 1.38X_3 - 1.11X_1X_2 + 0.41X_3 - 0.01X_2X_3 - 0.25X_1X_2X_3$$

The experimental values of *Fisher's ratio* are found to be 0.81 and 0.092 respectively, for *fired bending strength* and *hot permeability*. These values are lower than the tabulated value 6.09 of *Fisher's ratio.* Hence, the *non-linear* model is adequate.

The confidence intervals are found to be ±0.2269 and ±0.0736 for *fired bending strength* and *hot permeability* respectively. The significance of the regression coefficients in the *non-linear* model is decided from the confidence intervals. By retaining significant coefficients only, the final *non-linear* regression equations are:

$$\textit{Fired bending Strength} = 16.18 - 3.85X_1 + 7.68X_2 - 2.16X_3 - 0.75X_1X_2 - 0.99X_2X_3$$

$$\textit{Hot Permeability} = 12.03 + 4.25X_1 - 2.43X_2 + 1.38X_3 - 1.11X_1X_2 + 0.41X_3 - 0.01X_2X_3 - 0.25X_1X_2X_3$$

Effect of Linear and No-linear Regression

It can be seen from *Figs.* 3 and 4 that there is considerable difference between experimental mean values and linear regression values. The values of non-linear regression model as shown in figs. 5 and 6 are more repeatable and reproducible. The fitness of non-linear model is greatly improved due to the interaction of selected control *factors viz. grain size of filler, filler to binder ratio and stucco grit size*.

Effect of Grain Size of Refractory Filler

In planning operations the bending strength of shells improves significantly with the finer refractory filler in the slurry. This can be attributed to the better distribution and the increased surface area per unit volume of finer coal – flyash. The coarser cola-flyash produces high permeability and low strength. Coarse filler materials with greater void space have greater permeability than finer filler materials.

Effect of Filler: Binder Ratio in the Slurry

Filler: Binder ratio affects the character of the slurry and the shell under stable concentrations of the colloidal dispersion in the binder. The slurry becomes denser as Filler : Binder ratio increases. Thus, viscosity of the slurry and bending strength of the shells increase in proportion to the amount of filler added to the binder.

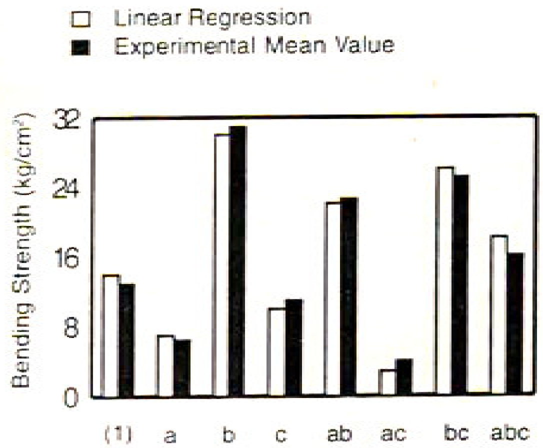

Fig. 3 : Effect of Linear Regression on Bending Strength

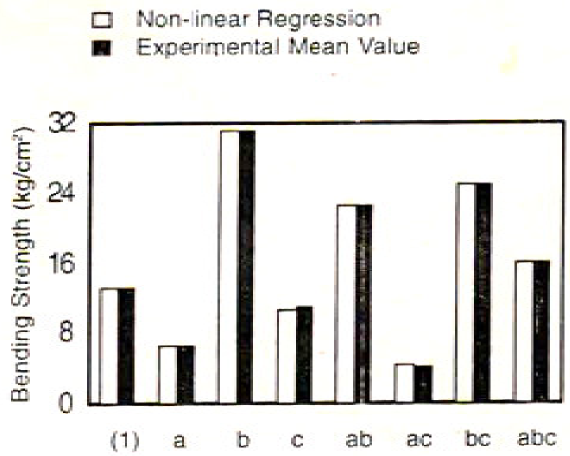

Fig. 5 : Effect of Non-linear Regression on Bending Strength

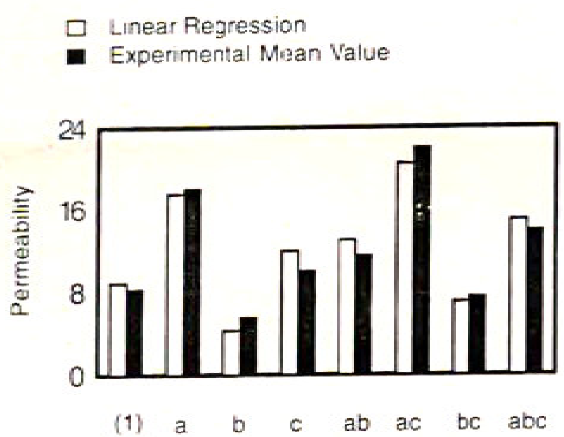

Fig. 4 : Effect of Linear Regression on Permeability

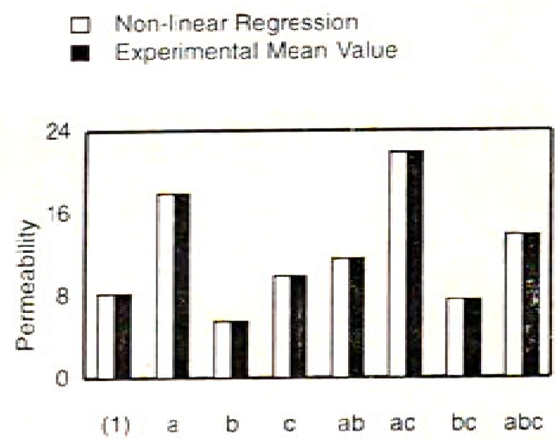

Fig. 6 : Effect of Non-linear Regression on Permeability

This is due partial neutralization of negative charges on silicon dioxide particles of binder by coal-flyash filler. The permeability of the shell decreases with increasing amount of filler in the slurry. This is because lesser number of voids are created in the shells with greater amount of filler in the slurry.

Effect of Stuccoing Grit Size

The finer the stuccoing grit, the thinner the shell. To achieve desirable shell thickness requires a large number of coatings. Shell bending strengths indicate that with coarse stuccoing grit, shell strength decreases. The finer sand grain size shows a very low permeability. In addition to average grain size, the grain size distribution has a pronounced effect on permeability.

CONCLUSION

A *third order non-linear regression model* proved to have an excellent fit in the case of Bending Strength and Permeability of shells. The Bending Strength of shells is increased by larger Filler : Binder ratio, finer filler in the slurry and finer stucco grit. The highest degree of improvement in the permeability of investment shell moulds is with coarse refractory filler and stuccoing sand the coal flyash can be used as a refractory filler material for the investment casting process as the strength and permeability values of shells are comparable with those shells made up of alumina, fused silica and zircon powder.

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APPENDIX – 1

Variance of optimization – using the definition[13]

$$S_Y{}^2 = \sum_{i=1}^{N}\sum_{j=1}^{n}\left(Y_{ij} - Y_i\right)^2/N(n-1)$$

Homogeneity of variance – using the definition[14]

$$G = \frac{Maximum\ variance\ of\ one\ of\ t\square e\ eig\square t\ treatments}{T\square e\ sum\ of\ all\ t\square e\ variances\ for\ every\ tratment}$$

$$= \frac{Largest\ S_i{}^2}{\sum_{i=1}^{N}\sum_{j=1}^{n} S_i{}^2(n-1)}$$

Where

N = number of treatments
n = number of replications
S_i $= Y_i - \bar{Y}_i$
Y_i = Response in the i^{th} treatment

APPENDIX – 2

Adequacy of regression model may be confirmed by Fisher's Ratio[15]:

$$F = S_{ad}{}^2/S_Y{}^2$$

Where

$S_{ad}{}^2$ = variance of adequacy

$= \sum_{i=1}^{N} n(Y_i - \bar{Y}_i)/f$

f = the degree of freedom

$= N - K - 1$ for a 2^k matrix

$N = 8$ and $K = 3$

APPENDIX – 3

The confidence interval, Δb_j for a given parameter may be written as

$$\Delta b_j = \pm tS_v/\sqrt{N}$$

Where
T = student's t at 5% significance level.
S_v = Square root of variance of optimization.

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