

NON-LINEAR FINITE ELEMENT MODELLING FOR DYNAMIC TRAJECTORY CONTROLLING OF ROBOTIC MANIPULATORS WITH FLEXIBLE LINKS

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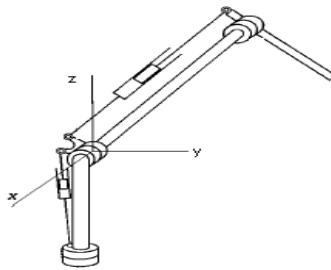
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ABSTRACT: The formulation of dynamic models for flexible spatial mechanisms such as robotic manipulators with six degrees of freedom have been formulated and evaluated. Interfaces for controls design with MATLAB and SIMULINK have been accomplished. A numerical simulation dealing with trajectory control of a flexible manipulator is presented.

1. INTRODUCTION

Flexibility effects increase as the weight of the robot manipulators decrease. It is desired to design lighter robots carrying out heavier payloads at higher speeds. Thus, the design of flexible link manipulators is a subject of intensive research. Designers of such manipulators need information about the dynamic response of the systems. Control actions can modulate the vibration effect of flexibility [1-12].

In this paper, a finite-element method is presented to formulate the dynamic equations for controlling a manipulator with flexible links. The equations of motion are formulated in terms of two sets of coordinates. One set represents the gross rigid link motion and the other represents the local flexible deformations. The method also permits generation of locally linearized models about a nominal trajectory. The nominal trajectory determines the position, velocity and acceleration of the manipulator mechanism with the restriction that all flexible deformations of the links are



suppressed.

Fig-1 The manipulator mechanism with six degrees of freedom

2. FINITE ELEMENT MODELING OF MANIPULATORS

The manipulator mechanism is modeled as an assembly of finite elements interconnected by joint elements such as hinge elements and truss elements as shown in Fig.1. Three different types of elements are used to model the manipulator of six degrees of freedom. The hydraulic cylinder is modeled as an active slider-truss element. The manipulator links are modeled by beam elements. There are also six cylindrical hinge elements, some of which are actuated by torque servos. Hinge elements are also used to model the wrist of the manipulator. The location of each element is defined relative to a fixed inertial coordinate system by a set of nodal coordinates x_i^j some of which may be Cartesian coordinates of the end nodes, while others describe the orientation of orthogonal triads, rigidly attached to the element nodes. The superscript j is added to show that a specific element j is considered. With respect to some reference configuration of the element, the instantaneous values of the nodal coordinates represent a fixed number of deformation modes for the element.

The deformation modes are specified by a set of deformation parameters e_1^j , some of which are associated with large relative displacements and rotations between the element nodes, while others describe small elastic deformations of the element and will be denoted by ϵ_1^j . The number of deformation parameters is equal to the number of nodal coordinates minus the number of degrees of freedom of the element as a rigid body. The components of the vector of deformation parameters e^j can be expressed as analytic functions of the vector of nodal coordinates x^j . A vector function $e^j = D^j x^j$ is defined for each element k . As an example of such a function the deformation function D^j of the slider-truss element is presented. The position of the slider-truss element is determined by the position vectors x^p and x^q of the end nodes p and q . A possible rotation of the element about the axis pq is not involved in the description of the element position. The number of degrees of freedom of the element as a rigid body is thus five, which gives rise to a single deformation parameter, associated with the elongation of the element. This elongation can be expressed as

$$e_1^j = D_1^j = ((x^p - x^q)^2 + (y^p - y^q)^2 + (z^p - z^q)^2)^{1/2} - l_0^j \dots(1)$$

where l_0^j is a reference length of the element. A proper definition of deformations requires that the deformation parameters e^j are invariant under rigid body movements of the element.

3. DEFINING EQUATIONS OF MOTION

The assemblage of finite elements is realized by defining a vector x of nodal coordinates for the entire mechanism. The deformation functions of the element can be described in terms of the components of x . The equations can be written as:

$$\begin{bmatrix} e_1 \\ \cdot \\ \cdot \\ \cdot \\ e_{n_e} \end{bmatrix} = \begin{bmatrix} D_1(x) \\ \cdot \\ \cdot \\ \cdot \\ D_{n_e}(x) \end{bmatrix} \dots(2)$$

$$e = D(x) \dots(3)$$

where n_e is the total number of deformation parameters of the mechanism. The kinematic constraints can be introduced by putting conditions on the nodal coordinates x as well as by imposing conditions on the deformation parameters e which are all assumed to be holonomic. For instance, if element k is rigid it has to satisfy the constraint equations $e^j = D^j(x^j) = 0$.

The motion of manipulator mechanisms is described by relative degrees of freedom, which can be either actuator joint coordinates, denoted e^k , as well as flexible deformation parameters denoted by ϵ^k . The objective of kinematic analysis is to solve equation (3) for the vector generalized coordinates $q = (e^m, \epsilon^m)^T$. The solution is expressed by means of a geometric transfer function F as

$$x = F(e^m, \epsilon^m) = F(q) \dots(4)$$

Generally this transfer function cannot be calculated explicitly from the constraint equations but has to be determined numerically in an iterative way [3].

The inertia properties of the concentrated and distributed mass of the elements are described with the aid of lumped and consistent mass matrices. For each element, the mass matrix M^j and the force vector f^j are defined. These give a contribution to the virtual power of $\langle (f^j - M^j \ddot{x}^j), \delta \dot{x}^j \rangle$. The loading state of each element is described by the stress resultant vector σ^j that is dual to \dot{c}^j . According to the principle of virtual power for the external forces including the inertial forces and stress resultant vector σ of the manipulator mechanism; the following relation is obtained:

$$\langle (f - M\ddot{x}), \delta \dot{x} \rangle = \langle (\sigma^{e,m}, \sigma^{\epsilon,m}), (\dot{e}^m, \dot{\epsilon}^m) \rangle \dots(5)$$

for all virtual velocities $\delta \dot{x}$, which satisfy all instantaneous kinematic constraints. By differentiating the transfer function (4), the following relation is obtained:

$$\dot{x} = \frac{\partial F}{\partial e^m} \dot{e}^m + \frac{\partial F}{\partial \varepsilon^m} \dot{\varepsilon}^m \quad \dots (6)$$

Using the differentiation operator D to represent partial differentiation with respect to the vector of the degrees of freedom, the following relation is obtained for Equation (6)

$$\dot{x} = DF.(\dot{e}^m, \dot{\varepsilon}^m) \quad \dots(7)$$

and for the second derivative

$$\ddot{x} = (D^2 F.(\dot{e}^m, \dot{\varepsilon}^m))(\dot{e}^m, \dot{\varepsilon}^m) + DF.(\ddot{e}^m, \ddot{\varepsilon}^m) \quad \dots(8)$$

Substitution of Equation (8) in the virtual power equation (5) yields the reduced equations of motion

$$\begin{bmatrix} \bar{M}^{ee} & \bar{M}^{e\varepsilon} \\ \bar{M}^{\varepsilon e} & \bar{M}^{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} \ddot{e}^m \\ \ddot{\varepsilon}^m \end{bmatrix} + \begin{bmatrix} D_{e^m} F^T \\ D_{\varepsilon^m} F^T \end{bmatrix} [M(D^2 F.(\dot{e}^m, \dot{\varepsilon}^m))(\dot{e}^m, \dot{\varepsilon}^m) - f] \\ + \begin{bmatrix} 0 & 0 \\ 0 & \bar{K}^{\varepsilon\varepsilon} \end{bmatrix} \begin{bmatrix} e^m \\ \varepsilon^m \end{bmatrix} = \begin{bmatrix} \sigma^{em} \\ 0 \end{bmatrix} \quad \dots(9)$$

with the reduced mass matrices:

$$\begin{aligned} \bar{M}^{ee} &= D_{e^m} F^T M D_{e^m} F \\ \bar{M}^{e\varepsilon} &= D_{e^m} F^T M D_{\varepsilon^m} F \\ \bar{M}^{\varepsilon e} &= D_{\varepsilon^m} F^T M D_{e^m} F \\ \bar{M}^{\varepsilon\varepsilon} &= D_{\varepsilon^m} F^T M D_{\varepsilon^m} F \end{aligned} \quad \dots(10)$$

The matrices $\bar{M}^{e\varepsilon}$ and $\bar{M}^{\varepsilon e}$ represent the dynamic coupling between the gross rigid motion and the flexible deformation of the links. The presence of this dynamic coupling is one of the major problems encountered in controlling flexible manipulators. The stress resultant vector of flexible elements is characterized by Hooke's law defined by $\sigma = K\varepsilon$, where K is a symmetric matrix containing the elastic constants. The driving forces and torques, represented by the vector σ^{em} , are applied only at the actuator joints; if the actuator dynamics are not considered then there is a simple linear relation between the vector of control inputs u and the vector σ^{em}

$$\sigma^{em} = -B.u \quad \dots(11)$$

where

$$B = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad \dots(12)$$

Equation (11) represents the input equation, and the matrix B is called the input matrix. The minus sign in Equation (11) is a result of different sign conventions for the driving forces in control engineering literature and in structural dynamics literature. With Equation (11), the equations of motion can be written in a more compact form

$$\bar{M}(q)\ddot{q} + DF^T [M(D^2 F.\dot{q}).\dot{q} - f] + \bar{K}q = Bu \quad \dots(13)$$

where \bar{M} is the system mass matrix and \bar{K} is the structural stiffness matrix of the manipulator mechanism.

4. LINEARIZED EQUATIONS OF MOTION

Given the non-linear equations of motion in Equation (13). Consider now small perturbations around the nominal trajectory $(q_0, \dot{q}_0, \ddot{q}_0)$ such that the actual variables are of the form

$$\begin{aligned} q &= q_0 + \delta q \\ \dot{q} &= \dot{q}_0 + \delta \dot{q} \\ \ddot{q} &= \ddot{q}_0 + \delta \ddot{q} \end{aligned} \quad \dots(14)$$

where the prefix δ denotes a first variation. The nominal values of the flexible deformation parameters and their time derivatives are assumed to be zero. Linearization of the reduced equations of motion (13) around the nominal trajectory results in

$$\bar{M}_0 \delta \ddot{q} + \bar{C}_0 \delta \dot{q} + (\bar{K}_0 + \bar{K}_0^N + \bar{G}_0) \delta q = B \delta u \quad \dots(15)$$

where \bar{M}_0 is the system mass matrix as in (13), \bar{C}_0 is the velocity sensitivity matrix, and \bar{K}_0 denotes the structural stiffness matrix as in (13), \bar{K}_0^N and \bar{G}_0 are the dynamic stiffness and the geometric stiffness matrix, respectively.

\bar{M}_0 , \bar{K}_0 and \bar{G}_0 are symmetric matrices, but \bar{C}_0 and \bar{K}_0^N need not. These matrices are calculated by [4].

$$\begin{aligned} \bar{C}_0 &= DF_0^T [D_x f DF_0 + 2MD^2 F_0 \cdot \dot{q}] \\ \bar{K}_0^N &= DF_0^T [D_x (f - M\ddot{x}_0) DF_0 + M (D^2 F_0 \cdot \ddot{q}_0 + (D^3 F_0 \cdot \dot{q}_0) \dot{q}_0) + D_x f D^2 F_0 \cdot \dot{q}_0] \\ \bar{G}_0 &= -D^2 F_0^T [f - M\ddot{x}_0] \end{aligned} \quad \dots(16)$$

The terms $\bar{C}_0 \delta \dot{q}$ and $\bar{K}_0^N \delta q$ are of importance for high-speed machinery. The term $\bar{G}_0 \delta q$ contributes most directly to the rigid link modes but tends to be quite weak for actively controlled manipulators. For the purpose of control system design of manipulator robots the matrixes \bar{C}_0 , \bar{K}_0^N and \bar{G}_0 may therefore be neglected. The resulting simplified linearised equations are of the form

$$\begin{bmatrix} \bar{M}_0^{ee}(t) & \bar{M}_0^{ee}(t) \\ \bar{M}_0^{ee}(t) & \bar{M}_0^{ee}(t) \end{bmatrix} \begin{bmatrix} \delta \dot{\epsilon}^m \\ \delta \ddot{\epsilon}^m \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \bar{K}_0^{ee}(t) \end{bmatrix} \begin{bmatrix} \delta \epsilon^m \\ \delta \epsilon^m \end{bmatrix} = \begin{bmatrix} I \\ 0 \end{bmatrix} \quad \dots(17)$$

The matrix coefficients of \bar{M}_0 are functions of time, since they depend on the nominal position, velocity and acceleration of the manipulator. For the purpose of the trajectory tracking control, the time-varying equations (17) may be converted in piecewise linearized time-invariant equations by dividing the total trajectory time T , in n intervals, where $t = t_i$, ($i = 1, 2, \dots, n$) denote the time at the end of interval i . The linear time is approximated by

varying system in the interval $\frac{(t_i + t_{i-1})}{2} < t < \frac{(t_i + t_{i+1})}{2}$ by the linear time-invariant system at $t = t_i$. The

linearized equations enable to determine the main natural frequencies and associated mode shapes for small motions of the manipulator about any point along the nominal trajectory. Two possible modal representations are of interest for control engineering design applications: namely, constrained modes and unconstrained or system modes. Constrained modes are determined from a modal analysis of the homogeneous system,

$$\bar{M}_0^{ee} \delta \ddot{\epsilon}^m + \bar{K}_0^{ee} \delta \epsilon^m = 0 \quad \dots(18)$$

which is obtained when the rigid link motion of the manipulator is fixed and the links vibrate about a certain prescribed configuration. Unconstrained or system modes are obtained from a modal analysis of the complete homogeneous part of Eq. (17). The simple structure of Eq. (17) enables to formulate a condition for the controllability of flexible manipulators in terms of these modal representations.

5. CONTROL SYSTEM DESIGN

In order to be able to design mechanical systems involving automatic controls (e.g. robotic manipulators) interfaces for control design with MATLAB (open-loop system analyses, Fig. 2) and SIMULINK (closed-loop simulations, Fig. 3) have been developed. These simulations use data from the open-loop analyses.

KIN is the kinematics module that analyzes the geometry of motion of the mechanism. The kinematic properties of the motion are specified by the geometric transfer functions.

DYN is the dynamics module that generates the equations of motion and performs numerical integration in the forward dynamic analysis.

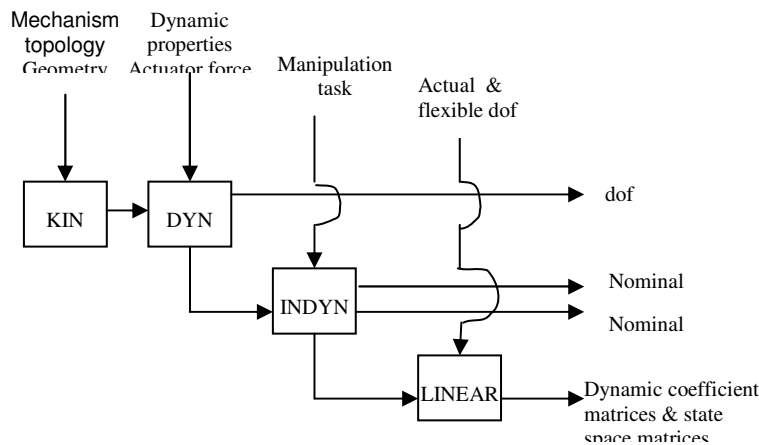


Fig-1 open loop system analysis

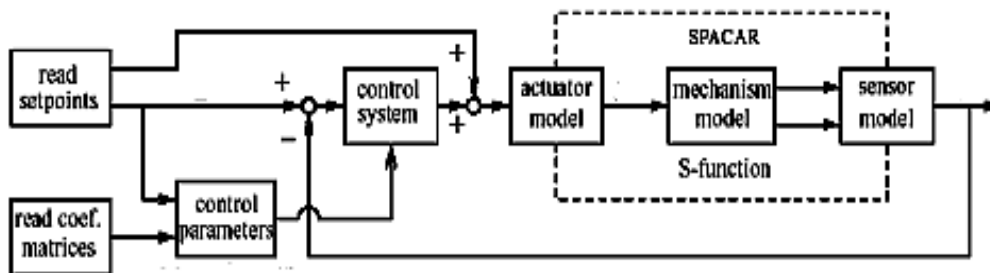


Fig-3 Closed loop system analysis

INDYDYN is the inverse dynamic analysis based on a rigid link model for the generation of the set points. The system outputs, represented by nominal output vector, y_0 consists of the coordinates to be monitored by control sensors. Coordinates that are not measured may be added to check the performance of the manipulator in the simulation. The system inputs, represented by the vector u_0 , are to be varied by control system actuators.

LINEAR is a forward dynamics stage for the generation of linearized equations and state space matrices. In case of a flexible manipulator additional generalized coordinates \mathcal{E}_i^m describing the elastic behavior of manipulator links can be added to the dynamic model.

The behavior of the manipulator mechanism with closed-loop control is simulated in SIMULINK as illustrated in Fig. 3. The nominal control input u_0 can be used as feed-forward compensation for the gross rigid link motion of the manipulator. The main part of the non-linear dynamic effects due to changes of the manipulator configuration is predicted by this nominal feed-forward compensation. The non-linear open-loop model of the manipulator with its actuators and sensors is into SIMULINK using a so-called S-function [4]. In this closed-loop simulation, the integration method is determined by the SIMULINK environment.

The output vector y is compared to the nominal output vector y_0 . The difference of these vectors is the input of the control system. The state matrices can be used to develop and tune a controller of any type (e.g. linear, nonlinear, discrete, continuous) by means of the available software tools in MATLAB and SIMULINK. The output of the controller δu is added to the nominal input vector u_0 to actuate the mechanism.

6. SIMULATION

6.1 System Properties

Fig-4 illustrates a finite element representation of the example manipulator. The revolute joints of the manipulator are modeled by the hinge elements (3), (5), and (7), where the hinge axes of the elements (5) and (7) are parallel, providing for the in-plane motion of the manipulator. The in-plane motion is driven by the linear actuators (1) and (2). The three major links of the manipulator are referred to as the shoulder (link 4), the upper arm (link 6) and forearm (link 8). The manipulator can rotate relative to the inertial reference frame (x, y, z) about the vertical axis of hinge element (3). This hinge element is driven by a torque source. The links (10) and (11) are rigidly connected with the upper arm. Referring to Fig. 4, Table1 lists the kinematic and dynamic properties of the links. The lumped mass of the bearing assembly and the end-effector are $m^3 = 10$ kg and $m^4 = 30$ kg, respectively. The gravity loads are evaluated as an applied force in the negative z direction at each mass, equal in magnitude to the weights of the masses. The upper arm and the forearm have uniform cross sections and are assumed to be flexible. The longitudinal deformation in the fore and upper arm and the torsional deformation in the forearm are suppressed. Each individual link is modeled by a single beam element. This corresponds to nine flexible degrees of freedom in addition to the actuator joint coordinates $e^1, e^2,$ and e^3 representing the elongation of the linear actuators (1) and (2) and the relative rotation of actuator (3). The actuators are modeled as pure force and torque sources without dynamics.

The manipulation task implies transferring the manipulator tip along a straight line with a trapezoidal velocity profile (Fig. 5). The initial and final manipulator configurations correspond with the configurations A and B.

6.2 Control Equations

In the type of actuator joint feedback used, proportional plus derivative feedback is utilized in which the position sensors are collocated with the actuators. The control law is given by

$$\delta u = -K_p \delta e^m - K_v \delta \dot{e}^m \quad \dots(19)$$

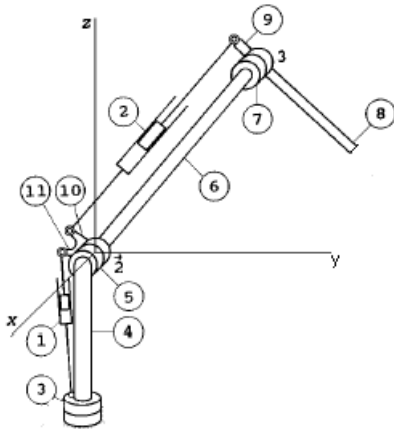


Fig-4 Finite modeling of the manipulator

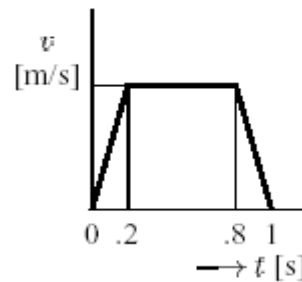


Fig-5 Velocity profile of the manipulator

where K_p and K_v are the position and rate feedback gain matrices. A straightforward way to achieve decoupling for the special case of rigid links is to specify the gain matrices K_p and K_v as

$$K_p = \hat{M}_0^{ee} \Omega^2 \quad \dots(20)$$

$$K_v = 2\bar{M}_0^{ee} \beta \Omega \quad \dots(21)$$

$$\Omega^2 = \begin{bmatrix} \omega_1^2 & & 0 \\ & \ddots & \\ 0 & & \omega_n^2 \end{bmatrix} \quad \dots(22)$$

$$\beta = \begin{bmatrix} \beta_1 & & 0 \\ & \ddots & \\ 0 & & \beta_n \end{bmatrix} \quad \dots(23)$$

The diagonal matrices Ω and β contain the desired servo loop frequencies and the corresponding active damping ratios. To obtain adequate damping of the lowest flexible mode, a bandwidth of approximately $\omega_i=0.5(\omega 1)_c$ and a relative damping $\beta_i \approx 0.7 \dots 0.9$ suffices, where $(\omega 1)_c$ is the lowest constrained natural frequency of the corresponding in-plane and out-of-plane motion.

7. SIMULATION RESULTS

The results of the nominal control synthesis for the motion trajectory of Fig. 5 are shown in Fig. 6. Because of the complex manipulator geometry, the individual joints undergo complex motions for the simple straight-line movements of the manipulator tip. As the inertia properties depend on the configuration, both the mass matrix \overline{M}_0^{ee} and the diagonal matrix Ω in the equations (20) and (21) for the gain matrices K_p and K_v must be updated during the simulation.

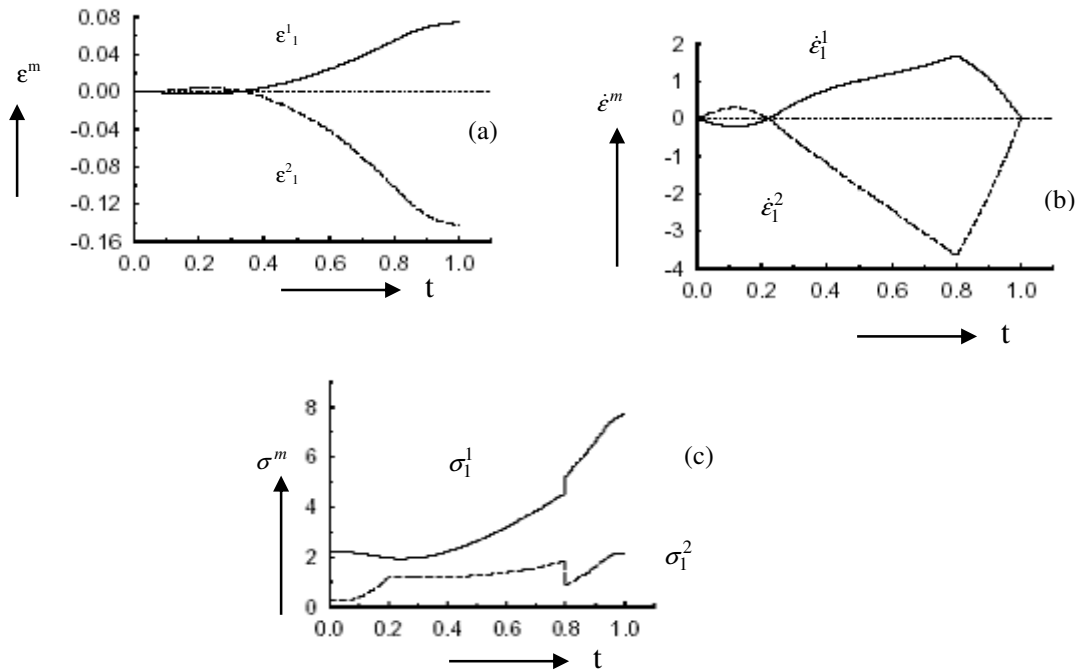


Fig-6 Positions (a), velocities (b) and driving forces (c) of the actuators (1) and (2)

The trajectory has been performed by taking 13 linearizations. The initial condition disturbances at the joint positions are determined by $\delta e^1 = 0$, $\delta e^2 = -0.01\text{m}$ and $\delta e^3 = -0.005\text{rad}$ corresponding with initial tip position deviations $\delta x^4 = -0.098\text{m}$, $\delta y^4 = -0.0022\text{m}$ and $\delta z^4 = -0.033\text{m}$ respectively.

8. CONCLUSIONS

The present finite element formulation has proved to be particularly useful for the numerical treatment of flexible manipulator analysis. This method simplifies conceptually the non-linear dynamic analysis involving large displacements and small (elastic) deformations. The geometric transfer function formalism provides a systematic approach for generating reduced non-linear dynamic equations and locally linearized manipulator models suitable for control system design. The MATLAB/SIMULINK yields a useful tool for controller design of geometrically non-linear engineering systems like mechanisms and manipulators. Control strategies can be developed using MATLAB, which is a standard tool for controller design. The dynamic behavior of the closed-loop system can be simulated using SIMULINK.

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