OPTIMUM TRAJECTORY PATH PLANNING FOR 3-DOF PLANAR ROBOT WITH MINIMUM JOINT DISTURBANCES

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Abstract: In this work, the explicit expressions for the joint disturbance torque of a 3 DOF planar redundant manipulator have been obtained. The dynamic programming method is employed to find the globally optimal joint trajectory to produce a straight-line motion that minimizes the joint disturbance torque over the entire motion. The resulting solution is compared with the solution obtained by minimizing the conventional joint torque, and it is shown that the joint motion obtained by the proposed method results in smaller joint disturbance torques.

Keyword: path planning, planar robot, joint disturbances

1. INTRODUCTION

Majority of the present day industrial robots are controlled by independent joint control schemes [1, 2]. The independent joint control schemes assume that a robot consists of independent linear joint control subsystem with fixed load inertia, rather than a multivariable control system. In the independent joint control schemes, the effects of varying effective inertia, Coriolis and centrifugal torques, and gravity are regarded as disturbance torque that is introduced into the control system [3]. This control scheme is easy, inexpensive, robust and reliable, and for this reason, widely used in industrial robot systems. The performance of this type of controller depends on the ability of the joint controller to reject the joint disturbance torque. This method gives a good performance when the joint speeds of the robot are low and the dynamic coupling between the joints can be ignored. However, as the motion of the joints become faster, the performance of this control scheme is degraded because the effects of the joint disturbance torque increase, and results in the joint trajectory tracking error [4-5]. Thus, for a high speed and high accuracy motion, it is important that the joint disturbance torque must be minimized at the motion planning stage.

A formulation and application of the joint disturbance torque for the independent joint controlled manipulator was reported in [6], where the joint disturbance torque was defined and used in the trajectory planning of a non-redundant manipulator such that the joint disturbance torque generated during the straight line motion was minimized. A robotic manipulator is said to be kinematically redundant when it has more joints than necessary for executing a task. For an n degrees of freedom manipulator in an m dimensional task space, if n > m, it is kinematically redundant. Since the joint space has greater degrees of freedom than the task space, for a given end effector location, there exist an infinite number of joint solutions. In order to select a joint solution from the infinite number of candidate solutions, thus resolving the kinematic redundancy, an optimization criterion is chosen and the joint solution that minimizes the optimization criterion is selected. Hence, by an appropriate choice of the optimization criteria, the redundant manipulator can be used to perform a secondary task. For example, the joint velocity minimization can be used as the optimization criterion to minimize the unwanted joint motion [7, 8], joint limit avoidance can be used to maintain the joint position near the center of the position range [9], the distance from obstacles can be used to execute a motion among obstacles without collision [10], singularity measures can be used to avoid singularities [11], kinetic energy can be used to minimize the power consumption, and joint torque minimization can be used to reduce the torque magnitude so that actuators with small torque bounds can be used to perform a heavy task.

The performance quality of the redundant manipulator, on the other hand, depends on the quality of the joint motion control. The optimization criteria mentioned above is concerned with the motion planning stage. No considerations are given to the joint control schemes, and the question of control performance is not addressed. When a manipulator is controlled by the independent joint control scheme, the performance is affected by the joint disturbance torques, and it is important that the joint disturbance torque be minimized at the motion planning stage. Hence, the kinematic redundancy can be exploited to reduce this joint disturbance torque.

In this paper, the minimization of joint disturbance torque is proposed as a new optimization criterion for the kinematic redundancy resolution when the independent joint control is employed. By minimizing the proposed criterion, the joint disturbance torque introduced to the independent joint control system is minimized at the motion planning stage, and as a consequence, the joint tracking error can be reduced during the actual motion. This is a motion-planning scheme suitable for high speed and high accuracy motion of the manipulator. The explicit expressions for the joint disturbance torque of a 3 DOF planar redundant manipulator are obtained. Then, dynamic programming method is employed to find the globally optimal joint trajectory to produce a straight-line motion that minimizes the joint disturbance torque over the entire motion. The resulting solution is compared with the solution obtained by minimizing the conventional joint torque.

2. JOINT DISTURBANCE TORQUE

The basis of the independent joint control is that a robot is viewed as a set of independent actuator-load pairs, where the load inertia of joints is unknown constants. The joint disturbance torque τ_{di}, are unknown and varies depending on the motion of the joints. For n joint robotic manipulator, the dynamic equation of motion can be expressed by the following Lagrange- Euler equation.

$$D(\theta)\ddot{\theta} + H(\theta,\dot{\theta}) = \tau \qquad (1)$$

where, D = (D_{ij}) is an nXn inertia matrix and H = (h_i) is an nX1 vector containing Coriolis, centrifugal effect and gravity loading vector, and τ is a n x 1 joint torque vector. If the robot manipulator is viewed as n independent joint control systems, the dynamic equation in (1) can be written as

$$\hat{D}_{ii}\ddot{\theta}_i + \tau_{di} = \tau_i \;, \quad i = 1, 2, 3, \ldots n \qquad (2)$$

where,

$$\hat{D}_{ii} = \text{sum of components of } D_{ii} \text{ which are not dependent on } \theta \qquad (3)$$

$$\tau_{di} = \sum_{j=1, j\neq i}^{n} D_{ij}(\theta)\ddot{\theta}_j + (D_{ii} - \hat{D}_{ii})\ddot{\theta}_i + h_i(\theta,\dot{\theta}) \qquad (4)$$

By definition, $\hat{D}_{ii}$ is not related with the joint position, velocity, and acceleration, and are constant regardless of the motion of the joints. $\hat{D}_{ii}$ is the constant inertial load for the joint i. In equation (4), τ_{di} is the disturbance torque of joint i and contains torque components in robot dynamic equation related to the variation of effective joint inertia due to joint position change, Coriolis and centrifugal effect, and gravity loading.

The joint disturbance torque in equation (4) is applied to a 3-DOF planar robot as shown in Fig.1. It is assumed that planar arm moves in a horizontal plane, and the gravity acceleration vector is set to zero. It is also assumed that the links have uniform mass distribution. In this figure, m_i, l_i, θ_i are mass, length and joint position of the i-th link respectively. The Lagrange-Euler equation of motion for this manipulator can be written in a below form.

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} \qquad (5)$$

and, from the definitions in equation (3) and (4), $\hat{D}_{11}, \hat{D}_{22}, \hat{D}_{33}, \tau_{d1}, \tau_{d2}, \tau_{d3}$ can be written as below:

$$\hat{D}_{11} = \frac{1}{3}m_1 l_1^2 + \frac{1}{3}m_2 l_2^2 + \frac{1}{3}m_3 l_{31}^2 + m_2 l_1^2 + m_3 l_1^2 + m_3 l_2^2 \quad (6)$$

$$\hat{D}_{22} = \frac{1}{3}m_2 l_2^2 + \frac{1}{3}m_3 l_3^2 + m_3 l_2^2 \quad (7)$$

$$\hat{D}_{33} = \frac{1}{3}m_3 l_3^2 \quad (8)$$

$$\begin{aligned}\tau_{d1} = &\begin{bmatrix} m_2 l_1 l_2 C_2 + 2m_3 l_1 l_2 C_2 + m_3 l_2 l_3 C_3 \\ + m_3 l_1 l_3 C_{23}\end{bmatrix}\ddot{\theta}_1 \\ &+ \begin{bmatrix}\frac{1}{3}m_2 l_2^2 + m_3 l_2^2 + \frac{1}{3}m_3 l_3^2 + \frac{1}{2}m_2 l_1 l_2 C_2 \\ + m_3 l_1 l_2 C_2 + m_3 l_2 l_3 C_3 + \frac{1}{2}m_3 l_1 l_3 C_{23}\end{bmatrix}\ddot{\theta}_2 \\ &+ \left[\frac{1}{3}m_3 l_3^2 + \frac{1}{2}m_3 l_2 l_3 C_3 + \frac{1}{2}m_3 l_1 l_3 C_{23}\right]\ddot{\theta}_3 \\ &- m_3 l_3 \dot{\theta}_1\dot{\theta}_3(l_2 s_3 + l_1 s_{23}) - m_3 l_3 \dot{\theta}_2\dot{\theta}_3(l_2 s_3 + l_1 s_{23}) \\ &- \frac{1}{2}\{m_3 l_3 \dot{\theta}_3^2(l_2 s_3 + l_1 s_{23})\} \\ &- \dot{\theta}_1\dot{\theta}_2 l_1(m_2 l_2 s_2 + 2m_3 l_2 s_2 + m_3 l_3 s_{23}) \\ &- \frac{1}{2}\{\dot{\theta}_2^2 l_1(m_2 l_2 s_2 + 2m_3 l_2 s_2 + m_3 l_3 s_{23})\}\end{aligned} \quad (9)$$

$$\begin{aligned}\tau_{d2} = &\begin{bmatrix}\frac{1}{3}m_2 l_2^2 + m_3 l_2^2 + \frac{1}{3}m_3 l_3^2 + \frac{1}{2}m_2 l_1 l_2 C_2 \\ + m_3 l_1 l_2 C_2 + m_3 l_2 l_3 C_3 + \frac{1}{2}m_3 l_1 l_3 C_{23}\end{bmatrix}\ddot{\theta}_1 \\ &+ [m_3 l_2 l_3 C_3]\ddot{\theta}_2 + \left[\frac{1}{3}m_3 l_3^2 + \frac{1}{2}m_3 l_2 l_3 C_3\right]\ddot{\theta}_3 \\ &- \frac{1}{2}\{m_3 l_2 l_3 \dot{\theta}_1\dot{\theta}_3 s_3 - \frac{1}{2}l_1\dot{\theta}_1^2(m_2 l_2 s_2 + 2m_3 l_2 s_2 + m_3 l_3 s_{23})\end{aligned} \quad (10)$$

$$\begin{aligned}\tau_{d3} = &\left[\frac{1}{3}m_3 l_3^2 + \frac{1}{2}m_3 l_2 l_3 C_3 + \frac{1}{2}m_3 l_1 l_3 C_{23}\right]\ddot{\theta}_1 \\ &+ \left[\frac{1}{2}m_3 l_3^2 + \frac{1}{2}m_3 l_2 l_3 C_3\right]\ddot{\theta}_2 \\ &+ \frac{1}{2}m_3 l_3(l_2\dot{\theta}_1^2 s_3 + 2l_2\dot{\theta}_1\dot{\theta}_2 s_3 + l_2\dot{\theta}_2^2 s_3 + l_1\dot{\theta}_1^2 s_{23})\end{aligned} \quad (11)$$

Let τ_d denotes the joint disturbance vector such that

$$\tau_d = \begin{bmatrix}\tau_{d1} \\ \tau_{d2} \\ \tau_{d3}\end{bmatrix} \quad (12)$$

$\hat{D}_{ii}$ in equations (6), (7) and (8) are components of D_{11}, D_{22} and D_{33} which are independent of the joint motion, and these can be used as the constant joint inertia in the independent joint controlled manipulators. Similarly, τ_{di} in (9), (10) and (11) are the components of the joint torques that depend upon the joint motion, and thus corresponds to the joint disturbance torques.

3. MINIMIZATION OF JOINT DISTURBANCE TORQUE

In this section, the joint disturbance torque formulated in section 2 is used in the kinematic redundancy resolution so that the optimal joint trajectory for a given end effector motion is obtained which globally minimizes the joint disturbance torque. The global optimization problems can be represented in the following general form in optimal control theory.

$$\begin{aligned}&\min \int P(z,u,t)d \\ &\text{such that} \\ &\dot{z} = s(z,u,t) \qquad \ldots(13) \\ &z^- \le z \le z^+ \\ &u^- \le u \le u^+\end{aligned}$$

where P is the optimization criterion, z is the system state, u is the control input.

The kinematic relations between the joint position θ and the task variable x of a manipulator can be expressed as follows:

$$\begin{aligned}x &= f(\theta) \\ \dot{x} &= J(\theta)\dot{\theta} \qquad (14) \\ \ddot{x} &= J(\theta)\ddot{\theta} + \dot{J}(\theta,\dot{\theta})\ddot{\theta}\end{aligned}$$

where x is the m - dimensional vector describing the end effector in task space, J represents the Jacobian matrix. In order to formulate the state equation, the dynamic equation of the manipulator is parameterized by (n-m) independent joint variables. Let the joint position and torque vectors be partitioned as

$$\theta = \begin{bmatrix}\tilde{\theta} \\ \bar{\theta}\end{bmatrix}, \qquad \tau = \begin{bmatrix}\tilde{\tau} \\ \bar{\tau}\end{bmatrix} \qquad 15)$$

where $\tilde{\theta}$ and $\bar{\theta}$ are dependent and independent joint variables respectively, and $\tilde{\tau}$ and $\bar{\tau}$ are the corresponding joint torques. The dependent joint variable $\tilde{\theta}$ can be given as $\tilde{\theta} = \tilde{\theta}(\bar{\theta}, x_d, t)$, a function of independent joint variable $\bar{\theta}$, and desired end effector path x_d through the kinematic relation (14). Let the Jacobian matrix be partitioned as $J = \left[\tilde{J}_{mxm} \ \bar{J}_{mx(n-m)}\right]$ then, the dependent joint velocities, $\dot{\tilde{\theta}}$ is given by $\dot{\tilde{\theta}} = \tilde{J}^{-1}\left(\ddot{x}_d - \bar{J}\dot{\bar{\theta}}\right)$. The state is chosen as $z \equiv \left[\bar{\theta}^T \dot{\bar{\theta}}^T\right]^T$, and the control input is chosen as $\bar{\tau}$. Then the state equation is given by

$$\frac{d}{dt}\begin{bmatrix}\bar{\theta} \\ \dot{\bar{\theta}}\end{bmatrix} = \begin{bmatrix}\dot{\bar{\theta}} \\ 0_{(n-m)xm} I_{(n-m)}\end{bmatrix} D^{-1}(\tau - H) \qquad (16)$$

In this work, n = 3 and m = 2, so there are two states and one control variable. The independent joint variable was selected as $\bar{\theta}$ = θ_3, hence the state variables used were z_1 = θ_3; z_2 = $\dot{\theta}_3$. The control variable was selected as $\bar{\tau} = \tau_3$, hence u = τ_3. The dynamic programming and technique [15] can now be applied to this optimal control problem.

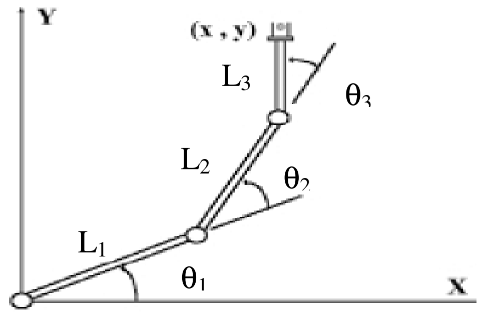

Fig-1 Three link planar manipulator

4. SIMULATIONS AND DISCUSSIONS

For the demonstration of kinematic redundancy resolution using global joint disturbance torque minimization, the three link planar robot in Fig.1 is used in this work. The link parameters used are l_1 = 1.0 meter, l_2= 0.8 meter, l_3 = 0.5 meter, m_1 = 10 Kg, m_2 = 8 Kg, and m_3 = 5 Kg. The manipulator task is to move from a initial end effector position of $i_{xd} = (1.94,0.83)^T$ with the joint position of $\theta_i = [0\ 30\ 30]^T$ degrees to the final end effector position of $f_{xd} = (1.36,\ 1.32)^T$, in a straight line trajectory. The position and velocity profile of the end effector along the straight-line path is shown in Fig.2. The joint velocities are zero at initial and final positions, and the end effector travels through a distance of 0.76 meters in 1.01 sec.

The optimization criterion used is the 2-norm square of the joint disturbance torque vector, P_1, formulated as follows. For comparison, the 2-norm square of joint torque vector, P_2 is also used.

$$P_1 = \|\tau_d\|^2 \qquad P_2 = \|\tau\|^2 \qquad (17)$$

Table-1: Step size parameters

	Lower limit	Upper limit	Step size
$Z_1 = \theta_3$	-0.5π radians	+0.5π radians	0.0021 radians
$Z_2 = \dot{\theta}_3$	-π rad/sec	+π rad/sec	0.063 rad/sec
u = τ_3	-10 NM	10 NM	0.2 NM
t	0 sec	1.01 sec	0.003 sec

For dynamic programming, the joint space was discretized as shown in Table-1. In order to reduce the memory requirement to the minimum, for each node, only the optimal control input from a node in a stage to the optimal node in the next stage was recorded. The cost of the node was not stored for the nodes in all stages. Instead, the cost of nodes of two stages, the present and next stages, were stored and used to compute the optimal control from present stage to the next stage. The total number of nodes was 4.443x10^8, each control input required 2 bytes memory storage, and the total memory requirement for the node data storage was 899.85 Mbytes. The data structures used for the node data are as below.

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short int control [14811][101][301];
double present Stage Cost [14811][101];
double next Stage Cost [14811][101];
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The optimal solution minimizing the joint disturbance torque, i.e. P = P_1, is shown in Fig. 3. The joint torques are shown in (a), and the joint disturbance torques are shown in (b). The resulting values of $P_1 = \|\tau_d\|^2$ and $P_2 = \|\tau\|^2$ are shown in (c). The resulting joint trajectory is shown in Fig.4. For comparison, the optimal solution minimizing the conventional joint torque, i.e. P = P_2, are shown in Fig. 5, and corresponding joint trajectory is shown in Fig.6. The simulation result shows that the peak value of joint disturbance torque, $P_1 = \|\tau_d\|^2$ shown in Fig.3(c) is an order of magnitude less than that shown in Fig. 5(c). The joint velocity of the two trajectories is shown in Fig. 7. This figure shows that minimizing the joint disturbance torque i.e. $P_1 = \|\tau_d\|^2$ results in a smaller joint motion. It can be seen that the kinematic redundancy can be used to reduce the joint disturbance torque while executing the specified end effector motion. The reduction of the joint disturbance torque is important when the joints are controlled by the independent joint control scheme, since the joint disturbance torque causes joint tracking error.

5. CONCLUSIONS

In an independent joint control scheme of a manipulator, the robot system is regarded as a set of independent actuator-load pairs, and the joint interaction such as centrifugal and Coriolis torques and the gravity effect are regarded as joint disturbance torque introduced into the joint control system. This joint disturbance torques results in the joint trajectory tracking error, and degrades the control performance. It is necessary to minimize the joint disturbance torque at the motion planning stage, and the kinematic redundancy of a manipulator can be exploited to reduce the joint disturbance torque. In this paper, the minimization of the joint disturbance torque is proposed as a new optimization criterion for the kinematic redundancy resolution when the independent joint control scheme is employed. Using a 3 DOF planar manipulator as an example, a globally optimal joint trajectory to produce a straight-line motion of the end effector is obtained that minimizes the joint disturbance torque during the motion. The result is compared with a solution obtained by minimizing the conventional joint torque, and it is shown that using the proposed criterion, a joint motion with reduced joint disturbance torque is possible.

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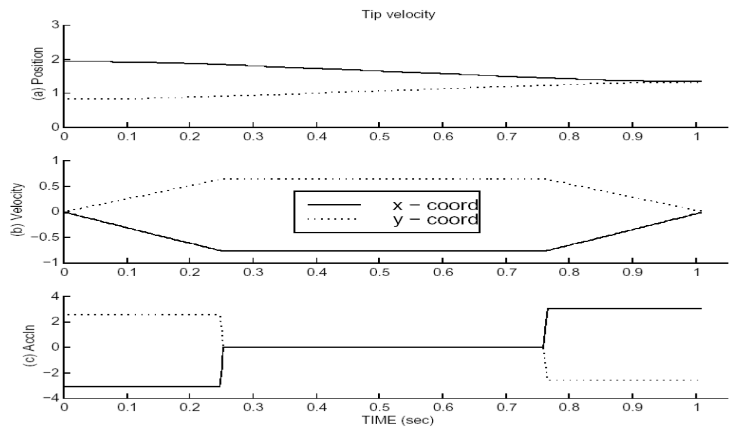

Fig.2 End effector trajectory along a straightline. (a) Position (b) Velocity(c) Acceleration

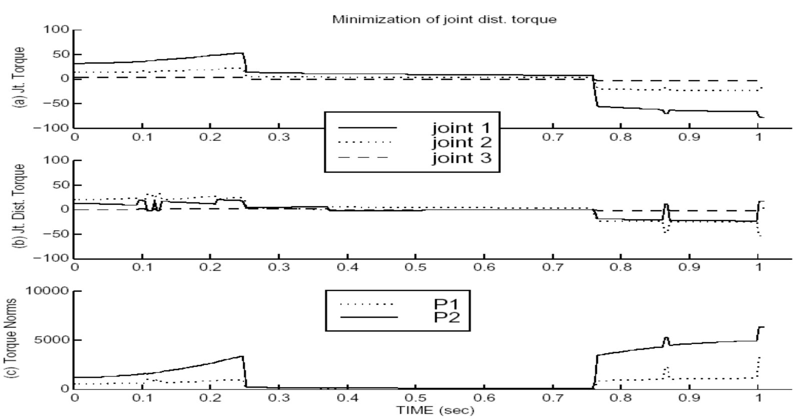

Fig.3 Joint disturbance torque minimization. (a) Joint torque (b) joint disturbance torque

(c) $P_1 = \|\tau_d\|^2$ and $P_2 = \|\tau\|^2$

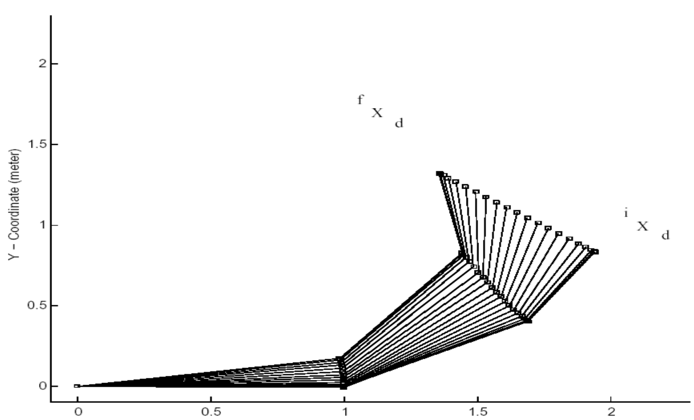

Fig.4 Motion trajectory for joint disturbance torque minimization

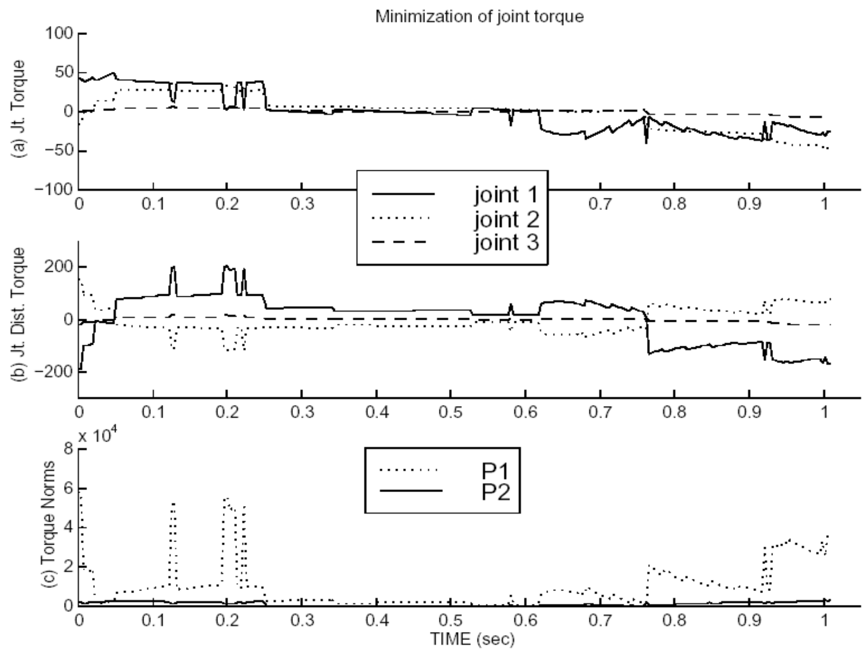

Fig.5 Result of joint torque minimization. (a) joint torque b) joint disturbance torque (c) $P_1 = \|\tau_d\|^2$ and $P_2 = \|\tau\|^2$

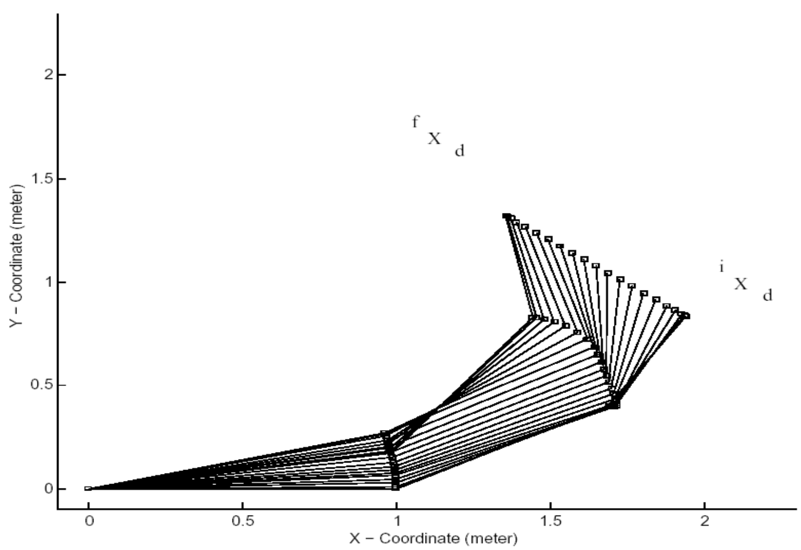

Fig.6 Motion trajectory for joint torque minimization.

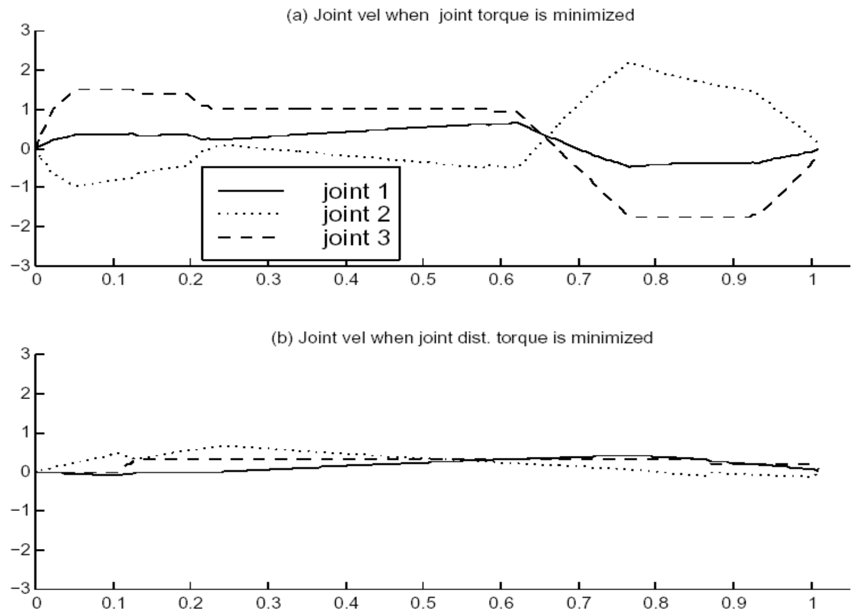

Fig.7 (a) joint velocity when $P_1 = \|\tau\|^2$ is minimized (b) joint velocity when $P_1 = \|\tau_d\|^2$ is minimized