

LARGE ROTATION AND LARGE DEFORMATION ANALYSIS OF FOUR-BAR MECHANISM

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Abstract: This paper presents the results of two finite element procedures, the corotational procedure and the absolute nodal coordinate formulation as applied to the four-bar mechanism. The limitations of the corotational procedure are characterized by motion discontinuities, high inertia forces and high and discontinuous angular velocities. It is shown that the incremental formulation can be less efficient and less accurate in large displacement problems as compared to the non-incremental absolute nodal coordinate formulation in which the nodal coordinates are defined in a fixed inertial frame.

1. INTRODUCTION

There are two types of finite element procedures that can be used for the large deformation analysis: incremental and nonincremental. The incremental approach is the most widely used procedure for the solution of non-linear large rotation and large deformation problems in structural applications. Several incremental procedures have been developed for nonisoparametric elements, in which infinitesimal rotations are used as nodal coordinates. Belytschko and Hsieh [1] used the convected coordinate system and applied it to the dynamic analysis of structural systems that undergo large rotations. A corotational procedure for the solution of nonlinear finite element large rotation problems was later proposed by Rankin and Brogan [2]. A new non-incremental approach, the absolute nodal coordinate formulation, has recently been proposed [3]. This formulation differs from other existing finite element formulations in the sense that no infinitesimal or finite rotations are used as nodal coordinates. Using this approach, beams and plates can be treated as isoparametric elements.

It is the objective of this paper to examine the performance of the absolute nodal coordinate formulation by comparing it with the corotational procedure and implemented in the finite element code ANSYS [4]. It will be shown that numerical problems are encountered when the incremental procedure is used in flexible multibody applications.

2. INCREMENTAL AND NON-INCREMENTAL FORMULATIONS

2.1 Incremental Finite Element Approach

In the incremental finite element formulation, nodal displacements and rotations are used as degrees of freedom [1, 2, 3, 5,6]. The internal forces of the flexible bodies are first defined in the element coordinate systems and then transformed to the global system. The dynamic equations are then solved for the deformation increments. The time/load steps are chosen such that the differences between two consecutive element configurations are small.

In the corotational procedure the contribution of the so-called large rigid-body rotations of the element is removed from the global displacement field through the use of an element convected coordinate system. A nonsingular large rotation vector, called pseudovector, is introduced to describe the nodal rotations. This pseudo vector was first introduced by Argyris [7, 8] and then slightly modified by Rankin and Brogan [2]. The successful implementation of the corotational procedure is due to the fact that it allows the use of the conventional finite element formulations. As pointed out by Rankin and Brogan [2], the corotational procedure is very accurate as long as the local rotations within the element remain less than 30°.

In the dynamics of flexible bodies that undergo large rotations, it is important to obtain accurate modeling of the inertia of the bodies. When the incremental formulations are used with consistent mass techniques, the global mass matrix of the element is not constant. As a consequence, the expression of the inertia forces does not take a simple form and these forces have to be updated at every time step. In the ANSYS code, the conventional shape function of the beam element is used. In this shape function, the axial displacement is approximated using a linear polynomial, while the transverse displacement is approximated using a cubic polynomial. This is the displacement field, which is used to generate the ANSYS results presented in the following sections.

2.2 Non-incremental Finite Element Approach (Absolute Nodal Coordinate Formulation)

In this formulation, the nodal coordinates of the element are defined in a fixed inertial coordinate system, and consequently no transformation is required for the element coordinates. The element nodal coordinates represent global nodal displacements and slopes. Thus, in the absolute nodal coordinate formulation, no infinitesimal or finite rotations are used as nodal coordinates and no assumption on the magnitude of the element rotations is made. It can be shown that the global shape function, that approximates both components of displacements using a cubic polynomial, contains a complete set of rigid body modes that can describe arbitrary rigid-body translational and rotational displacements. Hence using the absolute coordinates and slopes, it can also be shown that the beam element is an Isoparametric element. Since the mass matrix is constant, efficient and accurate numerical procedures can be used to solve the system of equations for the vector of the generalized accelerations.

3. FOUR BAR MECHANISM

The four bar mechanism shown in Fig.1, is considered for the representation of flexible multibody application. The dimensions and material properties of the links of the four bar mechanism are shown in Table-1. All components of the mechanism are made of steel and have a circular cross section with diameter equal to 0.4 m.

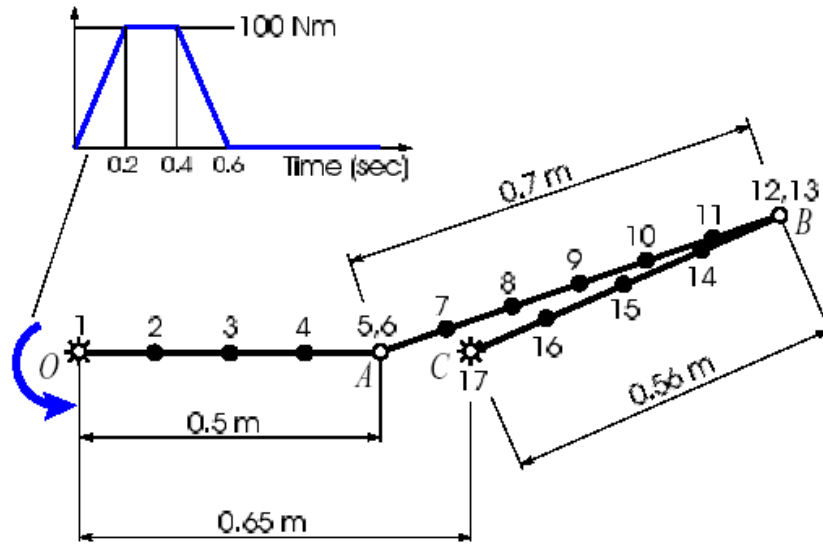


Fig.1 The four bar mechanism in the original configuration and the moment applied to the crankshaft versus time

Table-1: Parameters used in the simulation of the four-bar mechanism

Body	m (kg)	A (m ²)	I (m ⁴)	l (m)	E (Pa)
<i>Crankshaft</i>	4.9323	1.257E-03	1.257E-07	0.5	2.1E+11
<i>Coupler</i>	6.9052	1.257E-03	1.257E-07	0.7	2.1E+11
<i>Follower</i>	5.5242	1.257E-03	1.257E-07	0.56	2.1E+11

This system is designed to obtain high values of the angular velocities of the connecting rod and the follower as compared to the angular velocity of the crankshaft. In this system, complete rotations of the crankshaft are possible, as the Grashoff's law gives:

$$s + l = 1.2 \leq 1.21 = p + q$$

where s and l are the lengths of the shortest and longest links, and p and q are the lengths of the other two links. However, the difference between the two sides of the equation is very small, and this makes the motion non-smooth. When the rotation of the crankshaft is a multiple of 2π , the angular velocities of the connecting rod and the follower change dramatically in a very short time. The system is assumed to be driven by a moment, shown in Fig.1 as a function of time, applied to the crankshaft, and the effect of the gravity force is taken into consideration.

Several simulations have been performed using the corotational procedure implemented in ANSYS, using different numbers of steps in the integration routine. In the first simulation, 20,000 time steps were chosen while maintaining the option of automatic stepping active. This simulation configuration leads to the solution shown in Fig.2, where the global vertical position of point A on the crankshaft is presented and compared to the solution obtained using the absolute nodal coordinate formulation. Before 0.75 sec there is no difference between the two solutions, but after that the two curves diverge.

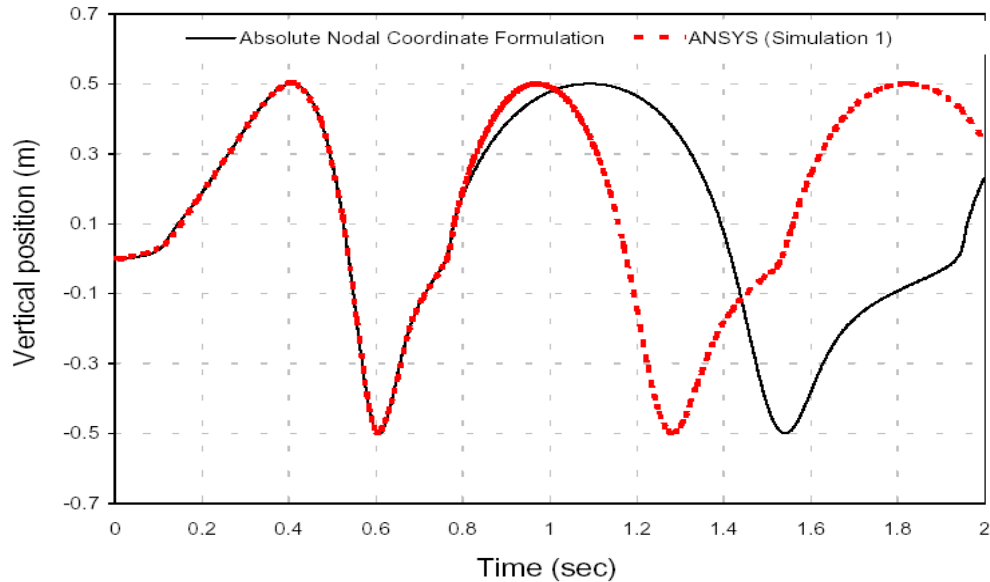


Fig.2 Global vertical position of point A on the crankshaft

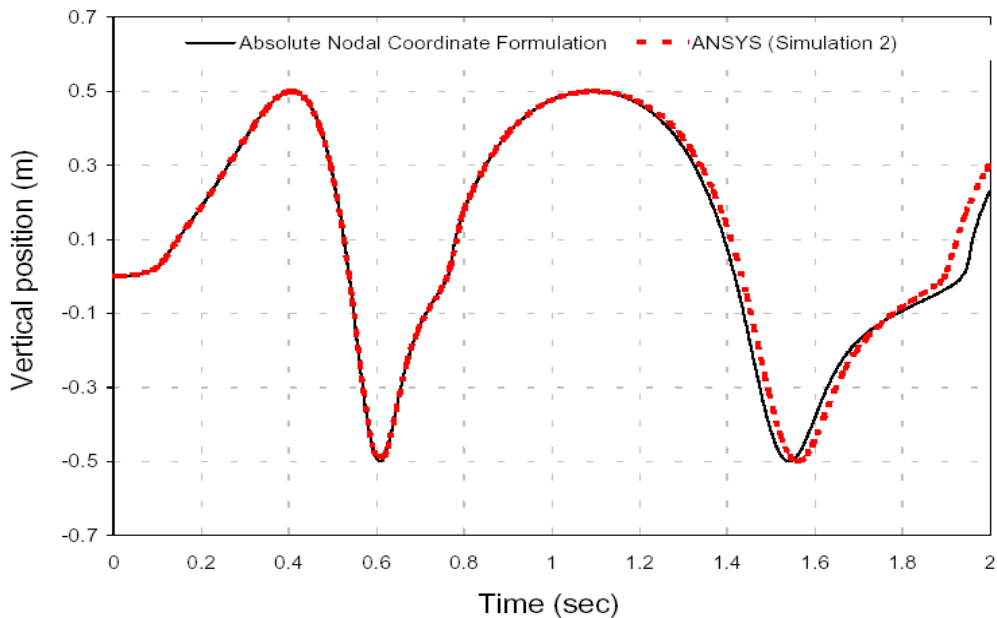


Fig.3 Global vertical position of point A on the crankshaft

It is clear that for the corotational procedure to converge, a smaller integration step is required. In order to achieve this, the automatic stepping option is removed in a second simulation. This change improves the results significantly, as demonstrated by the results shown in Fig.3. However, there are still differences when the deflections are considered instead of global positions of nodes, as demonstrated by the results shown in Fig.4. Furthermore, increasing the number of time steps to 40,000 does not lead to a better improvement of the results, as shown by the results of Fig.5.

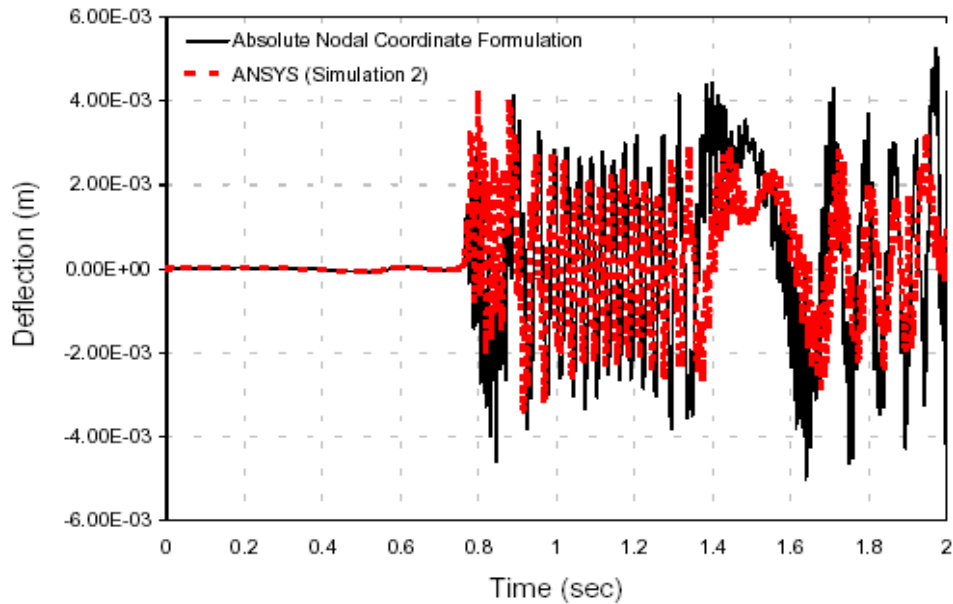


Fig.4 Transverse deflection of the midpoint of the connecting rod

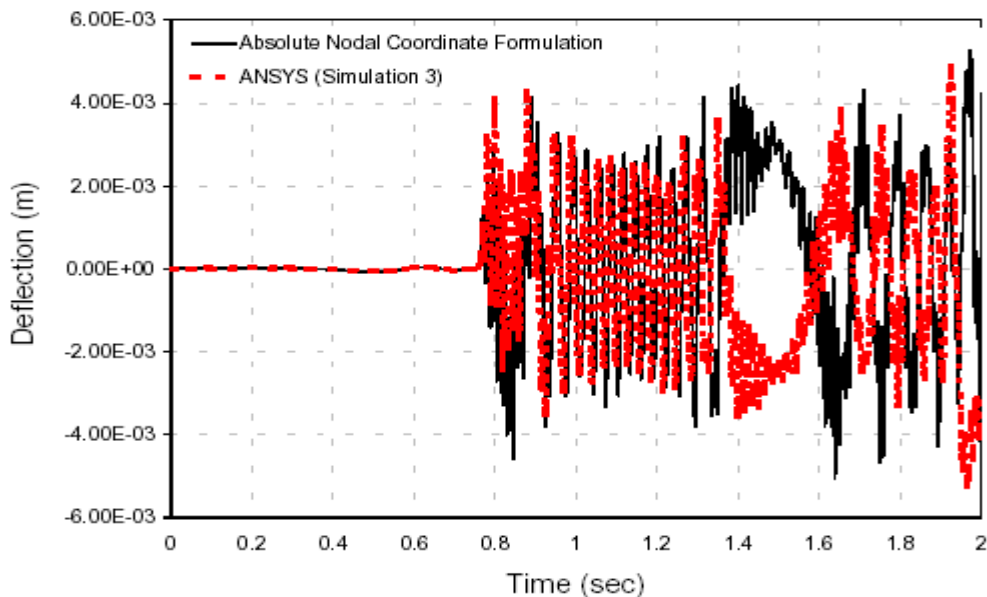


Fig.5 Transverse deflection of the midpoint of the connecting rod

4. CONCLUSIONS

In this investigation, the results of two finite element procedures, the corotational procedure and the absolute nodal coordinate formulation are compared. The limitations of the corotational procedure are demonstrated when flexible multibody applications characterized by motion discontinuities, high inertia forces and high and discontinuous angular velocities are considered. It is shown that the incremental formulation can be less efficient and less accurate in large displacement problems as compared to the non-incremental absolute nodal coordinate formulation in which the nodal coordinates are defined in a fixed inertial frame.

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