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Short communication

Spherical dome formation by transformation of superplasticity of titanium alloys and titanium matrix composites

J.J.V. Jeyasingh [a], G. Kothandaraman [a], P.P. Sinha [a], B. Nageswara Rao [b,*], A. Chennakesava Reddy [c]

[a] Mechanical Engineering Entity, Vikram Sarabhai Space Centre, Trivandrum 695022, India
[b] Structural Analysis and Testing Group, Vikram Sarabhai Space Centre, Trivandrum 695022, India
[c] Faculty of Mechanical Engineering, JNTU College of Engineering, Anantapur 515002, India

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Abstract

Titanium alloys and titanium matrix composites are useful materials in aerospace applications due to their high strength and stiffness, good corrosion resistance and low density. The gas pressure bulging of metal sheets has become an important forming method. As the bulging process progresses, significant thinning in the sheet material becomes obvious. This paper presents a simple analytical procedure for obtaining the dome height with respect to the forming time useful to the process designer for the selection of initial blank thickness as well as non-uniform thinning in the dome after forming. By thermally cycling through their transformation temperature range, coarse-grained, polymorphic materials can be deformed superplastically, owing to the emergence of transformation mismatch plasticity (or transformation superplasticity) as a deformation mechanism. This mechanism was examined under biaxial stress conditions during thermal cycling of titanium alloys with and without discontinuous reinforcements. For the transformation superplasticity, the strain-rate sensitivity index is considered as unity. The radius of curvature, thickness and height of the dome with respect to the forming time are obtained. The analytical results were found to be reasonably in good agreement with the test results.

Keywords: Titanium alloys; Titanium matrix composites; Transformation superplasticity; Spherical domes; Strain-rate sensitivity index; Forming pressure; Strain rate

1. Introduction

Superplastic forming has become a promising processing technique in manufacturing industry. Several models for bulge forming have been established [1–6]. Frary et al. [7] have investigated under biaxial stress conditions during thermal cycling of titanium alloys and composites. They used the CHIP process, which consists of blending of elemental metallic powders, cold isostatic pressing, vacuum sintering, and finally container less hot isostatic pressing to fabricate Commercial-Purity titanium (CP-Ti) and Ti–6Al–4V with and without discontinuous reinforcements. TiC particles (in the amount of 10 vol.%) were added to both CP-Ti and Ti–6Al–4V. These composites referred as Ti/TiC$_p$ and Ti–6Al–4V/TiC$_p$. Ti–6Al–4V reinforced with 5 vol.% TiB whiskers and referred the composite as Ti–6Al–4V/TiB$_w$ Disks of each material were machined from near-net-shape densified plates, with density greater than 99%. The disks had a diameter of 62 mm and a thickness ranging from 1.36 to 1.59 mm. The disk specimens were clamped into an Inconel pressurization vessel with an open die radius of 24 mm. The equipment for the performed biaxial dome experiments is shown in Fig. 1. It shows the central section of the quartz atmosphere tube, with a deformed specimen clamped in the pressurization vessel. It is also shown the range of the laser raster. Experiments were performed under gas pressure of 0.2 MPa and over the temperature range (840–970 °C) of the thermal cycle (frequencies: 15 h^{-1} for CP-Ti and 7.5 h^{-1} for Ti–6Al–4V and the respective composites). The transformation superplasticity constant in the constitutive relation is obtained from the measured dome height with respect to the forming time.

During gas-pressure dome bulging experiments, the dome height was measured as a function of time. It is noted from the test results of Frary et al. [7] that, the weakest material, CP-Ti deformed most rapidly, while the Ti–6Al–4V-based materials

* Corresponding author. Tel.: +91 471 2565831; fax: +91 471 2564181.
E-mail address: bnrao52@rediffmail.com (B.N. Rao).

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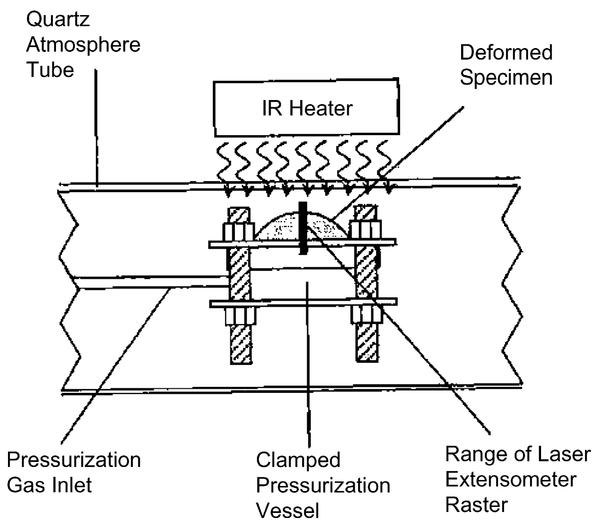

Fig. 1. Schematic representation of the experimental biaxial gas-pressure equipment.

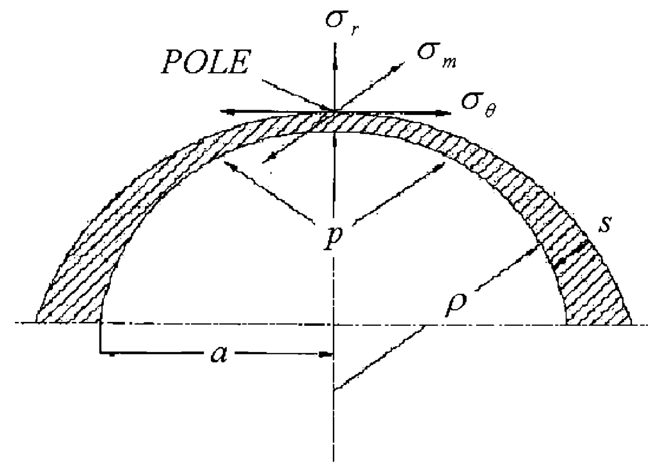

Fig. 2. Schematic representation of formed-dome using gas pressure forming.

deformed more slowly due to the longer thermal cycle times and higher creep resistance. However, during a thermal cycle, transformation superplasticity contributes to the deformation only when the phase transformation is occurring. At all other times, the material deforms only under the action of external stress by a typical creep mechanism (e.g. dislocation creep). Since these two mechanisms operate at different times during the cycle, they contribute to the total deformation independently, and it is reasonable to add their contributions:

$$\dot{\varepsilon} = \dot{\varepsilon}_{\text{creep}} + \dot{\varepsilon}_{\text{TSP}} = K_{\text{creep}}\sigma^n + K_{\text{TSP}}\sigma \tag{1}$$

where $\dot{\varepsilon}$ is the strain rate, σ the flow stress, n the stress exponent, K_{TSP} the transformation superplasticity constant and K_{creep} is the dislocation creep constant. The strain-rate sensitivity index (m) dependent on the flow stress for the constitutive relation (1) is:

$$m = \frac{\partial \ln \sigma}{\partial \ln \dot{\varepsilon}} = \frac{K_{\text{creep}}\sigma^n + K_{\text{TSP}}\sigma}{nK_{\text{creep}}\sigma^n + K_{\text{TSP}}\sigma} \tag{2}$$

It is noted from the experimental studies on biaxial deformation of composite materials that transformation superplasticity is the only operative deformation mechanism for the Ti–6Al–4V composites and there is no contribution from creep. Hence the constitutive relation for the present problem becomes

$$\dot{\varepsilon} = K_{\text{TSP}}\sigma \tag{3}$$

For transformation superplasticity, the strain-rate sensitivity index from Eq. (3) is found to be unity. This paper presents a simple analytical procedure for obtaining the dome height with respect to the forming time, which is useful to the process designer for the selection of initial blank thickness as well as to assess non-uniform thinning in the dome after forming.

2. Analysis

The solution for the biaxial dome formation through transformation superplasticity is presented below considering the incompressibility effects of isotropic material, power-law deformation and a spherical geometry having non-uniform thickness distribution. The diaphragm is rigidly clamped at the periphery. The thickness (s) of the specimen is very small compared with the die radius (a), so that bending and shearing effects are negligible and membrane theory is assumed. The coefficient K and the strain-rate sensitivity index (m) are constants in the constitutive equation: $\sigma = K(\dot{\varepsilon})^m$. The bulge surface shape keeps to that of a part of a sphere (see Fig. 2). There are three principal stresses at any point on the dome: the meridional stress (σ_m), the hoop stress (σ_θ), and the radial stress (σ_r, in the thickness direction). The value of σ_r is usually very small compared with σ_m or σ_θ and can be ignored. Then the stress state in the dome is $\sigma_m > 0$, $\sigma_\theta > 0$, $\sigma_r \approx 0$. A balanced biaxial stress state (i.e., $\sigma_m = \sigma_\theta$) exists at the dome apex.

Assuming plane stress, balanced biaxial stretching and volume constancy at the pole of the dome in free forming of a hemisphere, we can write the following relations:

$$\sigma_r = 0, \qquad \sigma_\theta = \sigma_m = \frac{p\rho}{2s}, \qquad \sigma_e = \sigma_\theta \tag{4}$$

$$\varepsilon_\theta = \varepsilon_m, \qquad \varepsilon_\theta + \varepsilon_m + \varepsilon_r = 0, \qquad \varepsilon_e = -\varepsilon_r \tag{5}$$

where p is the forming pressure; ρ the radius of curvature; σ_e the von Mises stress; and ε_e is the effective strain. ε_θ and ε_m are respectively the hoop and meridional strains. Thickness strain (ε_r) is compressive in nature whereas the effective strain (ε_e) should be considered as positive.

Taking into consideration the symmetry of a preform, let us consider bulging of one circular membrane with initial die radius, a. The initial blank thickness, s_0, is assumed to be small in comparison with the die radius, a (i.e., $s_0 \ll a$). The shape of the forming dome is a part of the sphere with the current radius, ρ. The material is assumed to be isotropic and incompressible, while flow stress depends on strain rate and temperature.

Let M be some point on the membrane, which at the initial moment of time ($t = 0$) belongs to the diameter AB (see Fig. 3). The envelope is clamped around its periphery and r_0 is the distance between point M and the centre of membrane O. At some current moment of time $t > 0$, point M goes to M′, while point O goes to point O′.

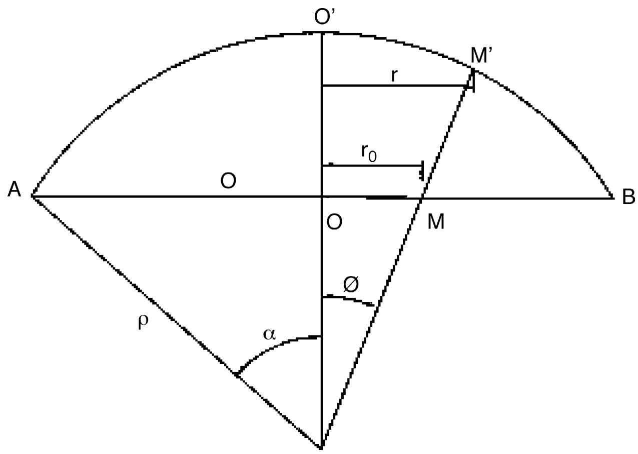

Fig. 3. Schematic representation of bulge forming of sheets.

The hoop, meridional and thickness strains at M′ are

$$\varepsilon_\theta = \ln\left(\frac{\rho\alpha}{a}\right) \tag{6}$$

$$\varepsilon_{\mathrm{m}} = \ln\left(\frac{2\pi r}{2\pi r_0}\right) = \ln\left(\frac{\rho\alpha}{a}\frac{\sin\phi}{\phi}\right) \tag{7}$$

$$\varepsilon_{\mathrm{r}} = \ln\left(\frac{s_\phi}{s_0}\right) \tag{8}$$

where s_ϕ is the thickness at M′. α is the angle subtended by the apex and the edge of the dome (see Fig. 3) and ϕ is the angle between the symmetry axis and the dome radius, drawing to the point under consideration.

Using Eqs. (6)–(8) in Eq. (5), we can obtain the variation in thickness as a function of position in the dome, which is independent of m:

$$\frac{s_\phi}{s_0} = \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\phi}{\sin\phi} \tag{9}$$

An empirical relation for the spherical dome thickness (s_ϕ) is [4,6]:

$$s_\phi = s_{\mathrm{p}} + (s_{\mathrm{e}} - s_{\mathrm{p}})\left(\frac{\phi}{\alpha}\right)^2 \tag{10}$$

Here s_{p} and s_{e} are thicknesses at the pole and edge of the dome.

A relation for s_{e} in terms of s_{p}, s_0 and m is [4]:

$$s_{\mathrm{e}} = \left[(1-C)s_0^{1/m} + Cs_{\mathrm{p}}^{1/m}\right]^m \tag{11}$$

where $C = \left(\frac{\sqrt{3}}{2}\right)^{1+(1/m)}$

Assuming volume constancy, we can write

$$\pi a^2 s_0 = 2\pi\rho^2 \int_0^\alpha s \sin\phi \, \mathrm{d}\phi \tag{12}$$

Using Eq. (10) in Eq. (12), we can obtain

$$s_0 = 2\,\mathrm{cosec}^2\,\alpha[s_{\mathrm{p}} I_0 + (s_{\mathrm{e}} - s_{\mathrm{p}})I_1] \tag{13}$$

where $I_0 = 1 - \cos\alpha$ and $I_1 = -\cos\alpha + (2\sin\alpha/\alpha) - ((2/\alpha)\sin(\alpha/2))^2$

Using Eq. (11) in Eq. (13), we can obtain a non-linear equation for s_{p}. For the specified value of 'm', the thickness at the pole (s_{p}) can be determined by solving the resulting non-linear equation through Newton Raphson's iterative method. s_{p}/s_0 can be represented after solving Eq. (13) in the form:

$$\frac{s_{\mathrm{p}}}{s_0} = \left(\frac{\sin\alpha}{\alpha}\right)^2 f(\alpha) \tag{14}$$

where $f(\alpha)$ is a polynomial function of α. The coefficients in the polynomial function $f(\alpha)$ are dependent on the strain-rate sensitivity index (m).

Assuming the applied pressure (p) as independent of time and using Eqs. (3), (4) and (14), we can write the following equation for α:

$$\frac{\mathrm{d}\alpha}{\mathrm{d}t} = -K_{\mathrm{TSP}}\left(\frac{p\rho}{4s_0}\right)\left(\frac{\alpha^3}{\sin^2\alpha}\right) \times \left\{(\alpha\cos\alpha - \sin\alpha)f(\alpha) + \frac{1}{2}\alpha\sin\alpha f'(\alpha)\right\}^{-1} \tag{15}$$

Initial condition for Eq. (15):

$$\alpha = 0 \quad \text{at} \quad t = 0 \tag{16}$$

Here, $f'(\alpha)$ denotes differentiation of the polynomial function, $f(\alpha)$ with respect to α.

For a very small forming time, Eqs. (15) and (16) give:

$$\alpha = \left\{\frac{9}{4}K_{\mathrm{TSP}}\left(\frac{pa}{s_0}\right)t\right\}^{\frac{1}{3}} \tag{17}$$

The height of the dome from the solution of α is:

$$h = \frac{a}{\sin\alpha}(1 - \cos\alpha) \tag{18}$$

3. Results and discussion

Eq. (13) is solved for the pole thickness of the dome (s_{p}) specifying different values of the strain-rate sensitivity index (m) and the angle, α. From the results, the unknown function, $f(\alpha)$ in Eq. (14) is expressed as a fourth-order polynomial function of α:

$$f(\alpha) = 1 + a_1\alpha + a_2\alpha^2 + a_3\alpha^3 + a_4\alpha^4 \tag{19}$$

The coefficients a_1, a_2, a_3 and a_4 in the polynomial function are dependent on the value of strain-rate sensitivity index (m). Table 1 gives the coefficients for different values of m. Dutta and Sharma [8] presented the optimum temperature, strain rate, flow-stress and strain-rate sensitivity for the titanium alloy (Ti–6.3Al–2.7Mo–1.7Zr) were: 1173 K, $3.3 \times 10^{-4}\,\mathrm{s}^{-1}$, 7.06 MPa and 0.85. It is noted from their experimental results that a hemisphere of diameter 30.3 mm was formed, by blowing a 2.7 mm thick blank by argon gas. The thicknesses of the dome were measured at every 10° interval by using a point micrometer. The pole thickness estimated from Eqs. (9) and (14) are: 1.094 and 1.145 mm, respectively, whereas the measured thickness at the pole reported in Ref. [8] is 1.194 mm. Predictions on pole thickness based on Eq. (14) will be comparable to the measured

Table 1
Coefficients a_1, a_2, a_3 and a_4 in the fourth-order polynomial function, $f(\alpha)$ in Eq. (19) for different values of the strain-rate sensitivity index, m

Strain-rate sensitivity index (m)	Coefficients in the polynomial function, $f(\alpha)$			
	a_1	a_2	a_3	a_4
0.20	0.0023	−0.0396	0.0638	−0.1249
0.25	0.0179	−0.1095	0.1987	−0.1692
0.30	0.034	−0.1741	0.2967	−0.192
0.35	0.0388	−0.1843	0.311	−0.1824
0.40	0.0352	−0.1582	0.2749	−0.1558
0.45	0.0301	−0.1253	0.2295	−0.1284
0.50	0.0247	−0.0945	0.1883	−0.1053
0.55	0.0202	−0.0681	0.1529	−0.086
0.60	0.0169	−0.0485	0.1271	−0.0718
0.65	0.0134	−0.0299	0.1035	−0.0595
0.70	0.0117	−0.019	0.0895	−0.0513
0.75	0.0096	−0.0075	0.0754	−0.0438
0.80	0.0086	−0.0003	0.0662	−0.0384
0.85	0.0075	0.0067	0.0575	−0.0336
0.90	0.0067	0.0119	0.0513	−0.0299
0.95	0.0059	0.0166	0.0458	−0.0268
1.00	0.0055	0.0196	0.0425	−0.0245

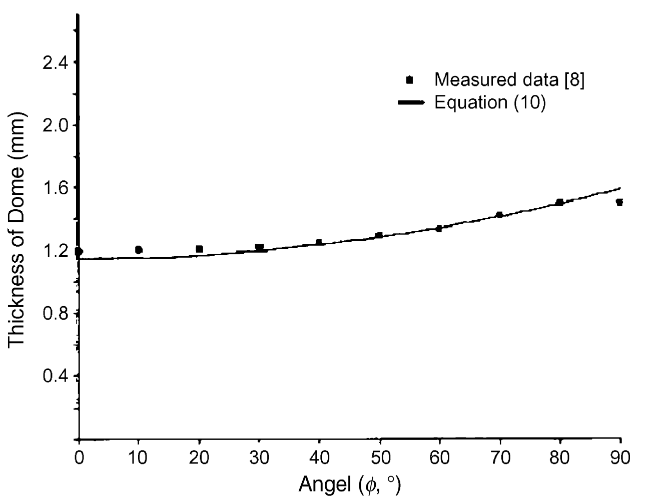

Fig. 4. Comparison of thickness variation from pole to the edge of the dome made of titanium alloy (Ti–6.3Al–2.7Mo–1.7Zr).

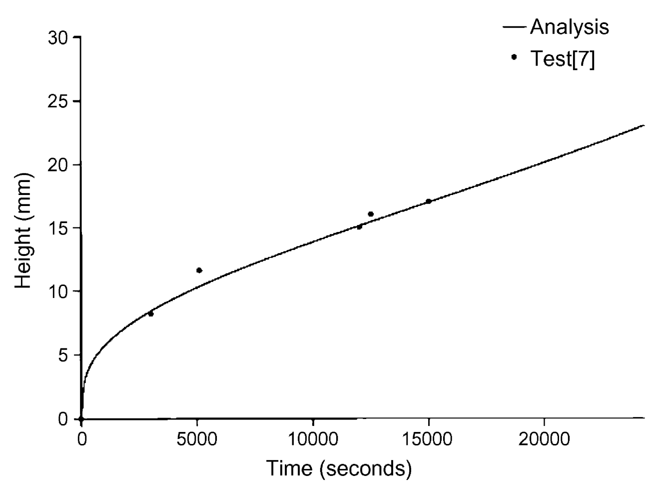

Fig. 5. Variation of dome height with time for Commercial-Purity, CP-Ti.

values. Hence, the pole thickness (s_p) estimated from Eq. (14) is used in Eq. (11) for the evaluation of the thickness at the edge of the dome (s_e). Using these values in Eq. (10) and specifying the angle ϕ between 0 and α, we can get the variation of thickness from pole to the edge of the dome. Fig. 4 demonstrates the closeness between the simulated and the measured values.

From the biaxial experiments, Frary et al. [7] reported the transformation superplasticity constant, K_{TSP} for titanium alloys and composites. Using this value in Eq. (15) and solving for α, the dome height (h) with respect to the forming time can be obtained from Eq. (18). In the present analysis, Eq. (15) is solved by the finite difference method with a fixed step-size, $\Delta t = 100$ s. Specifying the values of ϕ between

Table 2
Comparison of analytical and experimental pole thickness of spherical domes after superplastic forming

Material	K_{TSP} (10^{-12} Pa^{-1} s^{-1})	Test [7]			Present analysis		
		S_0 (mm)	S_p (mm)	ε_e (Eq. (20))	α (°)	S_p (mm)	ε_e (Eq. (20))
CP-Ti	9.58	1.390	0.636	0.782	87.50	0.638	0.779
Ti/TiC$_p$	4.20	1.360	0.759	0.583	77.25	0.758	0.585
Ti–6Al–4V	2.60	1.391	0.878	0.460	69.50	0.858	0.483
Ti–6Al–4V/TiC$_p$	2.54	1.510	1.179	0.247	52.00	1.179	0.247
Ti–6Al–4V/TiB$_w$	2.40	1.590	1.372	0.147	40.50	1.372	0.147

Strain-rate sensitivity index, $m = 1$; die radius, $a = 24$ mm; forming pressure, $p = 0.2$ MPa.

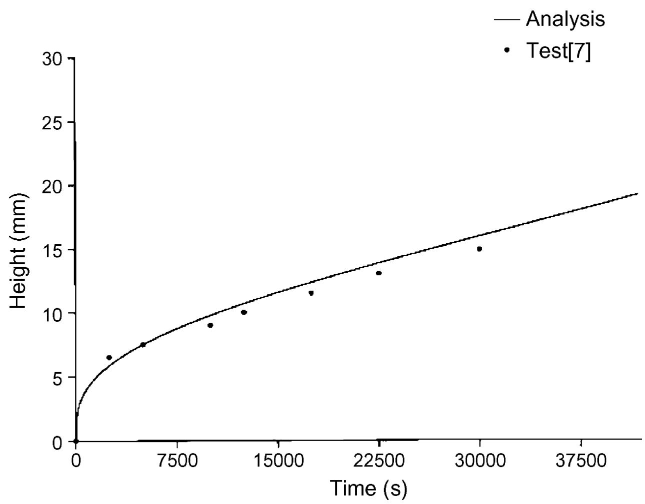

Fig. 6. Variation of dome height with time for titanium matrix composite, Ti/TiC_p.

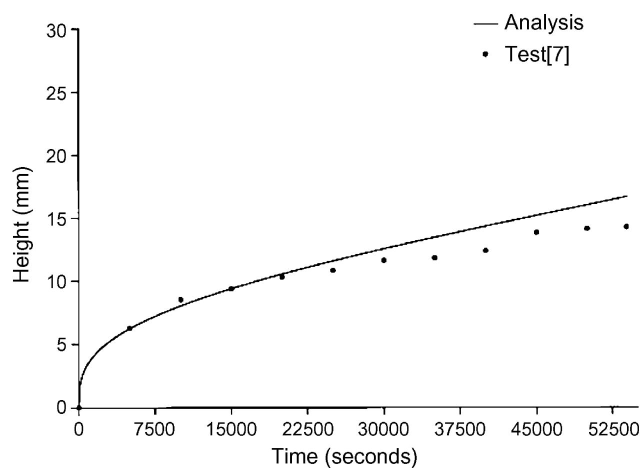

Fig. 7. Variation of dome height with time for titanium alloy, Ti–6Al–4V.

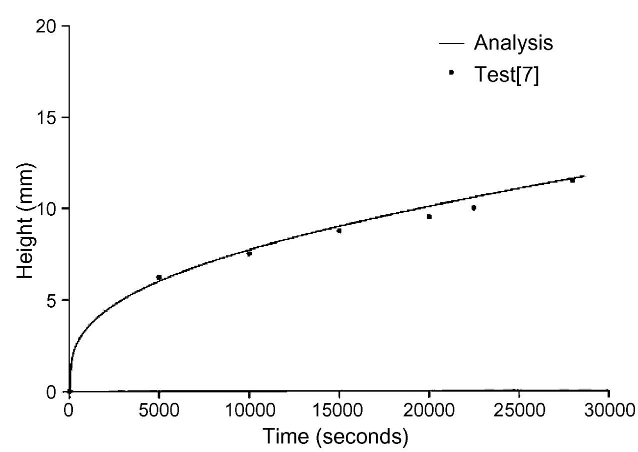

Fig. 8. Variation of dome height with time for titanium matrix composite, Ti–6Al–4V/TiC_p.

0 and α, the thinning of the dome can be obtained from Eqs. (10), (11) and (14). Table 2 gives comparison of analytical and experimental results related to the pole thickness of domes made of titanium alloys and composites. The von

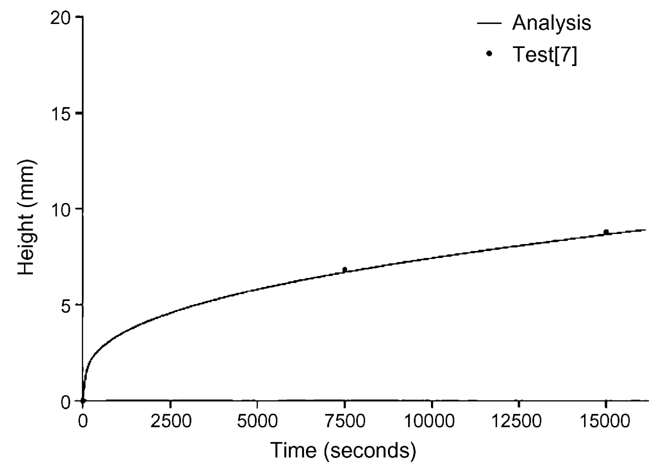

Fig. 9. Variation of dome height with time for titanium matrix composite, Ti–6Al–4V/TiB_w.

Mises' equivalent strain at the pole of the dome is obtained from

$$\varepsilon_e = -\varepsilon_r = \ln\left(\frac{s_0}{s_p}\right) \tag{20}$$

Figs. 5–9 show the comparison of analytical and experimental results of dome height (h) with respect to the forming time for titanium alloys and composites. The analytical results are found to be reasonably in good agreement with test results.

4. Concluding remarks

The solution for the biaxial dome formation through transformation superplasticity is presented considering the effects of incompressibility in the isotropic material, power-law deformation and a spherical geometry having non-uniform thickness distribution. A simple numerical procedure is described for obtaining the dome height with respect to the forming time. This procedure is validated comparing the analysis results with the existing biaxial dome experimental results of titanium matrix composites. Though the formulation of the problem is related to the constant pressure forming, it can be extended easily for the case of gas pressure forming of domes under constant strain rate, in which the forming pressure varies with time.

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