Two-Dimensional Theoretical Modeling of Anisotropic Wear in Carbon/Epoxy FRP Composites: Comparison with Experimental Data

A. Chennakesava Reddy* and M. Vidya Sagar

Professor of Mechanical Engineering
JNT University College of Engineering (Autonomous), Hyderabad, India
*E-mail: dr_acreddy@yahoo.com

Abstract

A two-dimensional anisotropic wear model is constructed to predict the anisotropic wear of composites based on the anisotropic strength and the contact behavior. Compared with the experimental data, the predicted results for the wear rate in the transverse orientation demonstrate a reasonable accuracy with the experimental results. The predicted wear rate in the parallel orientation is slightly larger than in the transverse orientation, which is inconsistent with the experimental result. The two-dimensional model is sufficient to predict anisotropic wear in the transverse orientation of fibers.

Keywords: two-dimensional modeling, anisotropic wear, carbon/epoxy.

Introduction

Many tribological components such as brakes, clutches, driving wheels, bolts, nuts, gears, cams, and bearings are used in the mechanical machinery. The friction and wear appearing, while functioning of these components, is one of the largest energy losses [1]. The commonly used materials for these components range from metals, alloys, ceramics, polymers, and composites [2]. A unidirectional continuous fiber-reinforced polymer (FRP) is a class of tribological materials that possess self-lubricating capability and low noise.

The most general fibers of FRP composites are E-glass, carbon, and aramid. E-glass fibers are formed from calcium aluminoborosilicate. Aramid is produced using para-phenylene terephthalamide. Carbon or graphite fibers are currently the best known and most widely used. The purpose of matrix material in the manufacturing of FRP composites is to bind the fibers together. The cohesive and adhesive
characteristics of matrix materials give the ability of transferring load to and between fibers, and safeguard the composites from environmental conditions and handling. The majority of matrix materials are epoxy, polyether ether keton (PEEK), and polyphenylene sulfide (PPS).

Unidirectional continuous fiber-reinforced polymer composites reveal significant tribological anisotropy due to their heterogeneity. Fiber orientations encompass a significant influence on the wear and friction performance of FRP composites [3]. Experimental explorations have shown that the major wear resistance in FRP composites occurred when the sliding was normal to the fiber orientation, while the lowest wear resistance occurred when the fiber orientation was in the transverse direction. Experiments have also publicized that the coefficient of friction and the wear in FRP composites depend on several factors including the material combination, the fiber orientation, and the surface roughness. The wear phenomena in the carbon/epoxy and Kevlar/epoxy composites were enlightened using a delamination theory [4]. In this work, the wear was found to be related to the stress field under the indentation of an asperity, the mechanics of the crack nucleation and propagation, and the material properties. A series of experiments were conducted to ascertain the influence of fiber orientation, elastic modulus, loading conditions, friction coefficient, interlaminar shear strength, and the fracture strain on the wear rate of FRP composites [5]. Based on the experimental results, the following empirical wear rate equation was anticipated:

$$\dot{w} = \rho \left( \frac{fp}{E} \right)^n \frac{1}{I_s}$$

where,
- $\rho$ is the wear constant,
- $n$ is an exponential parameter,
- $f$ is the friction coefficient,
- $p$ is the applied pressure,
- $E$ is the elastic modulus, and
- $I_s$ is the interlaminar shear strength.

By means of the empirical wear equation (1), the tribological properties of unidirectional polyphenylene sulfide-carbon fiber laminate composites were established to qualitatively explain the cause of fiber orientation in terms of difference in the interlaminar shear strength and the fracture strain of the three principal fiber orientations [6]. In order to theoretically explain the influence of fiber orientation on the wear of composites, a relationship was established between the wear rate of normally oriented FRP composites and the fiber debonding depth under the indentation of a spherical asperity [7]. For the wear of FRP composites in the parallel orientation, a model of a beam lying on a foundation to simulate the fibers in a polymer matrix was constructed [8]. These models clearly express the wear modes in the three principal directions and the influence of the microstructures of FRP composites on the wear.
Two-Dimensional Theoretical Modeling of Anisotropic Wear

Objectives
This paper is to build up wear model of unidirectional continuous FRP composites. The unidirectional continuous FRP composites are modeled as a quasi-homogeneous, transversely isotropic elastic half-space that is in contact with an infinitely long, rigid parabolic cylinder. A Cartesian coordinate system is defined such that the Y-axis coincides with the vertical axis of the cylinder as shown in Figure 1. The sliding direction, v, is perpendicular to the axis of cylinder, and motion occurs from left to right (-a, b) is the interval of the contact patch.

![Figure 1](image1.png)

**Figure 1:** Contact model of a rigid cylinder on FRP composite.

![Figure 2](image2.png)

**Figure 2:** Principal fiber orientations.

**Table 1:** Mechanical properties of carbon/epoxy FRP composites.

<table>
<thead>
<tr>
<th>FRP</th>
<th>Fiber Elastic Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Matrix Elastic Modulus (GPa)</th>
<th>Poisson’s Ratio</th>
<th>Composite Frictional Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon + Epoxy</td>
<td>230</td>
<td>0.25</td>
<td>0.33</td>
<td>0.35</td>
<td>0.20</td>
</tr>
</tbody>
</table>
Table 2: Strength properties of carbon/epoxy FRP composites.

<table>
<thead>
<tr>
<th>FRP</th>
<th>Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbon + Epoxy</td>
<td>Longitudinal tensile strength (MPa)</td>
</tr>
<tr>
<td></td>
<td>1375</td>
</tr>
</tbody>
</table>

When the coordinates coincide with the principal axes of the elastic half-plane (as shown in Figure 2), the stress-strain relationship of uni-directional composites can be sufficiently described using five elastic constants. In such a condition the X-Y plane is considered transversely isotropic, and the generalized Hook’s law becomes:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\
c_{12} & c_{22} & c_{23} & 0 & 0 & 0 \\
c_{13} & c_{23} & c_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & c_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & c_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & c_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{31} \\
\gamma_{12}
\end{bmatrix}
\]

The mechanical and strength properties of carbon/epoxy FRP composites are given Table 1 and 2.

Development of Wear Model

Wear is closely associated with failure stresses for isotropic materials because the wear process involves plastic yielding and deformation. It has been recapitulated that there are three main contributions to wear: adhesion, abrasive, and asperity deformation [9]. Abrasive and adhesive wear are the two principal mechanisms for most ductile isotropic materials. For these materials, the wear rate may be expressed in the form [10]:

\[
\dot{W} = k F \frac{F}{H}
\]

where, k is the abrasive/adhesive wear constant factor; F is the applied force; and H is the material hardness. k, describing the probability that the material loses as wear debris, is dependent on materials. The hardness, defined as the applied load divided by the projected indentation area, is a measure of the plastic deformation in the contact region. The relationship between the hardness and the yield strength for isotropic materials has been given by the equation [11]:

\[
H = 2.8 \sigma_y
\]
The yield stress $\sigma_y$ in equation (3) may be obtained according to von Mises shear strain energy criterion or Tresca’s plastic criterion. Equations (2) and (3) demonstrate the quantitative relationship between wear and the failure strength. The adhesive and abrasive wear mechanisms have been found in anisotropic FRP composites in the same way [3]. The wear of FRP composites, but, cannot be characterized directly in terms of Equations (2) and (3) because the von Mises yield criterion cannot be applied to FRP composites due to the anisotropy. In addition, the equations (2) and (3) do not include the influence of the fiber orientation on the wear. As observed on their worn surfaces [5], the wear modes of fiber-reinforced composites entail fiber fracture, fiber pullout, matrix crack, and fiber and matrix thinning. With the exception of the fiber and matrix thinning, all of the above wear modes are similar to the micro-scale failure modes of the fibrous composites. Hence, it is practical to assume that the anisotropic failure criteria can be employed to characterize the wear of fiber-reinforced composites.

The application of a macro-scale anisotropic failure criteria implicitly requires that the wear of FRP composites satisfy two hypotheses: (1) the composites are homogeneous and (2) the contact and wear behavior of composites results from the averaged or mixed constituent material properties in a direction. In general, the above two assumptions are well conceived because the wear performance largely depends upon the bulk mechanical properties of materials. In this case, FRP composites may be assumed quasi-homogeneous and anisotropic. The Tsai-Wu failure criterion is a quadratic tensor polynomial criterion that includes linear terms [12].

The relationship between the stresses in the principal material coordinate system (Figure 3) and those in the global coordinate system is:

$$\sigma_1 = \sigma_x \cos^2 \theta + \sigma_y \sin^2 \theta + 2\tau_{xy} \sin \theta \cos \theta$$  \hspace{1cm} (4)

$$\sigma_2 = \sigma_x \sin^2 \theta + \sigma_y \cos^2 \theta - 2\tau_{xy} \sin \theta \cos \theta$$  \hspace{1cm} (5)

$$\tau_{12} = -\sigma_x \sin \theta \cos \theta + \sigma_y \sin \theta \cos \theta + 2\tau_{xy} \cos 2\theta$$  \hspace{1cm} (6)

where $\sigma_1$ and $\sigma_2$ are principal stresses and $\theta$ is the orientation of fibers.

![Figure 3: Illustration of principal stresses in the FRP composites.](image-url)
From the equations (4) to (6), the influence of friction on the failure strength can be abstracted. When the composites are under the compressive shear stresses, the friction shear stress is:

\[ \tau_{xy} = f\sigma_x \]  

(7)

where \( f \) is the coefficient of friction.

Here, the stress in the horizontal direction of the composites is not considered to elucidate the effect of friction (\( \sigma_y = 0 \)).

Wear rate is a function of loading conditions, material properties, and interface characteristics. An empirical wear rate equation should not only curve-fit the experimental data at reasonable accuracy, but also elucidate the wear phenomena. In this work, an anisotropic wear rate equation is constructed based on the following two physical criteria:

1. In a fiber orientation, the greater the fracture strength, the less the wear.
2. In a fiber orientation, the greater the stress, the more the wear.

Using the above two conditions as a hypothesis, the following first-order anisotropic wear model is proposed:

\[ \dot{w} = k \frac{P_{\text{max}}}{\sigma_x} \]  

(8)

Where, \( \dot{w} \) is the wear rate,
\( k \) is a composite material wear factor;
\( P_{\text{max}} \) is the maximum contact pressure varying with fiber orientations;
\( \sigma_x \) is the failure compressive stress.

The physical meaning of this model represents the probability that material loss occurs. In the equation (8), the maximum contact pressure substitutes the applied load in the original abrasive or adhesive model equation (2). The maximum contact pressure is used because the contact behavior of fiber-reinforced composites is anisotropic and the same loading may lead to different contact stress levels in different fiber orientations. In general, the maximum contact pressure in the normal orientation is several times that in the transverse orientation. Based on the equation (8), the wear of fiber-reinforced composites can be theoretically predicted and investigated.

Compared to equations (1) and (2), the wear model in equation (4) offers the following favorable features:

1. It is an anisotropic and includes anisotropic stress and strength.
2. The friction condition is implicitly included in the strength.
3. The contact behavior is expressed in terms of the maximum contact pressure, which is a function of the directional elastic properties of composites and the external loading. equation (1) only considers a single Young’s modulus of the composites.
Figure 4: Effect of fiber orientation on the compressive strength.

Comparison Theoretical Data with Experimental Data

In figure 4, it is exposed that the coefficient of friction has significant influence on the failure strength when the fiber orientation is near to 90°. It indicates that the failure mechanism at the normal orientation of fibers is governed by the compressive when the friction coefficient is small; when the friction coefficient is large, the failure mechanism is dominated by the shear strength and transverse compressive strength. In the empirical wear rate equation (1), the wear is assumed to be proportional to the friction coefficient. Here, the friction loading is implicitly included in the failure strength as a nonlinear term. With the relationships between the strength and the fiber orientations, an anisotropic wear model may be constructed for the fiber-reinforced polymer composites.

A theoretical predication of wear for carbon/epoxy is exercised as a function of the fiber orientation as it varies from the transverse to the normal orientation. It is important to note that the composite material in the model is in a plane stress state as shown in Fig.1. The elastic constants are transformed as below:

\[ c_{ij} = C_{ij} - C_{13} C_{j3} / C_{33} \]  

(9)

Since the wear factor is dependent on materials, it can be used to construct a method to verify the wear model in equation (8) because \( k \) should be constant for a specific material. According to equation (8), the wear factor is:

\[ \dot{w}_{test} = k \left( \frac{P_{max}}{\sigma_s} \right) \]  

(10)

In equation (10), the wear factor equals to the ratio of the test wear data over the theoretical values, \( \dot{w} \left( \frac{P_{max}}{\sigma_s} \right) \) . If equation (8) accurately predicts the wear rate for
a specific material, the ratios of test wear data over the theoretical values, \( \tilde{\omega} \left( \frac{P_{\text{max}}}{\sigma_d} \right) \), should be equal for different fiber orientations, i.e.:

\[
k_1 = k_2 = \ldots = \text{const} \tan t
\]  

(11)

Therefore, in order to examine the above relationship (equation 11), the wear factors for carbon/epoxy are shown in figure 5.

![Figure 5: Variation wear factor with fiber orientation in the carbon/epoxy FRP Composites.](image)

![Figure 6: Comparison of theoretical with experimental wear rate in the transverse orientation of carbon/epoxy composites.](image)
The trend of predicted wear rate in the transverse orientation is invariable with the experiment results as shown in figure 6. The trend of predicted wear rate in the parallel orientation is larger than the experimental results as sighted in figure 7. The predicted wear rate in the parallel orientation is slightly greater than in the transverse orientation (figure 8), which is inconsistent with the experimental results as observed in figure 9. The experimental wear in transverse orientation was 1.4 times that in parallel orientation [4]. This indicates that a two-dimensional plane stress model is not sufficient to distinguish the wear difference between the parallel and the transverse orientations. A three-dimensional geometrical model is required to accommodate the difference of the shear stress strength among three principal material orientations for highly anisotropic materials.
Conclusion
In this investigation, anisotropic strength theories are employed to analyze the wear of FRP composites. A two-dimensional anisotropic wear model is constructed to predict the anisotropic wear of composites based on the anisotropic strength and the contact behavior. Compared with the experimental data, the predicted results for the wear rate in the transverse orientation of fibers exhibit a reasonable accuracy.

References


