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DIFFERENCE BETWEEN DENAVIT - HARTENBERG (D-H) CLASSICAL AND MODIFIED CONVENTIONS FOR FORWARD KINEMATICS OF ROBOTS WITH CASE STUDY

A. Chennakesava Reddy
Professor of Mechanical Engineering
JNTUH College of Engineering
Kukatpally, Hyderabad

Abstract: This paper highlights the difference between the D-H classical convention and D-H modified convention for RR manipulator. The forward kinematics ends at a frame, whose origin lies on the last joint axis z_2 as per modified D-H convention, a_2 does not appear in the link parameters whereas a_2 appears in the link parameters of D-H classical convention.

Keywords: RR manipulator, D-H classical convention, D-H modified convention

Address all correspondence to: e-mail: dr_acreddy@yahoo.com

1. KINEMATIC CHAIN

A robotic manipulator may be considered as set of links connected in a chain called **kinematic chain** by joints (figure 1). The simple joints are prismatic joint (figure 2a) and revolute joint (figure 2b). The prismatic joints permit a linear motion and the revolute joints allow a rotary motion. The revolute and prismatic joints exhibit one degree of freedom (dof).

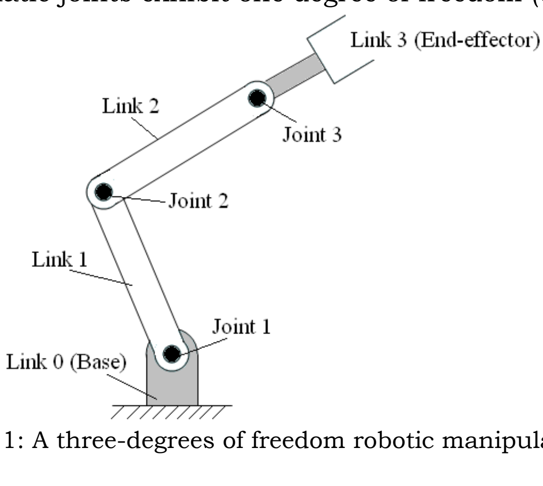

Figure 1: A three-degrees of freedom robotic manipulator

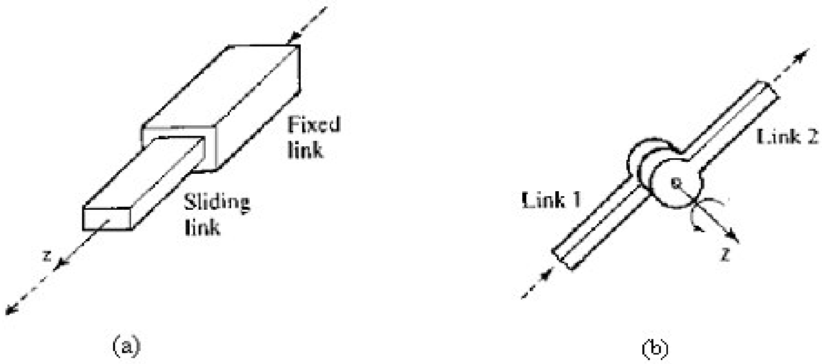

Figure 2: Joints (a) Prismatic joint, and (b) revolute joint

Typical robots are *serial-link manipulators* comprising a set of bodies, called *links*, in a chain, connected by *joints*1. In this book, each joint has one degree of freedom, either translational or rotational. Note that the assumption does not involve any real loss of generality, since joints such as a ball and socket joint (two degrees-of-freedom) or a spherical wrist (three degrees-of-freedom) can always be thought of as a succession of single degree-of-freedom joints with links of length zero in between. For a manipulator with *n* joints numbered from 1 to *n*, there are *n*+1 links, numbered from 0 to *n*. Link 0 is the base of the manipulator, generally fixed, and link *n* carries the end-effector. Joint *i* connects links *i* and *i* -1. when joint *i* is acutated, link *i* moves. A link can be specified by two numbers, the *link length* and *link twist*, which define the relative location of the two axes in space. Joints may be described by two parameters. The *offset length* is the distance from one link to the next along the axis of the joint. The *joint angle* is the rotation of one link with respect to the next about the joint axis.

A coordinate frame is attached rigidly to each link. To facilitate describing the location of each link we affix a coordinate frame to it: frame *i* is attached to link *i*. When the robotic manipulator executes a motion, the coordinates of each point on the link are constant. Each joint has a joint axis with respect to which the motion of joint is described. By convention, the *z*-axis of a coordinate frame is aligned with the joint axis.

Description of an end-effector in space requires a minimum of six degrees of freedom. Typical robotic manipulators have five or six degrees of freedom. The objective of forward kinematic analysis is to determine the cumulative effect of the entire set of joint variables.

The displacement of joint is denoted by q_i and is called joint variable. The collection of joint variables

$$q = [q_1, q_2, ..., q_n]^T \quad (1)$$

is called the joint vector.

The position of the end-effector is denoted by the dimensional vector

$$r = [r_1, r_2, \ldots, r_m]^T \quad (2)$$

The relation between *r* and *q* determined by the manipulator mechanism is given by

$$r = f(q) \quad (3)$$

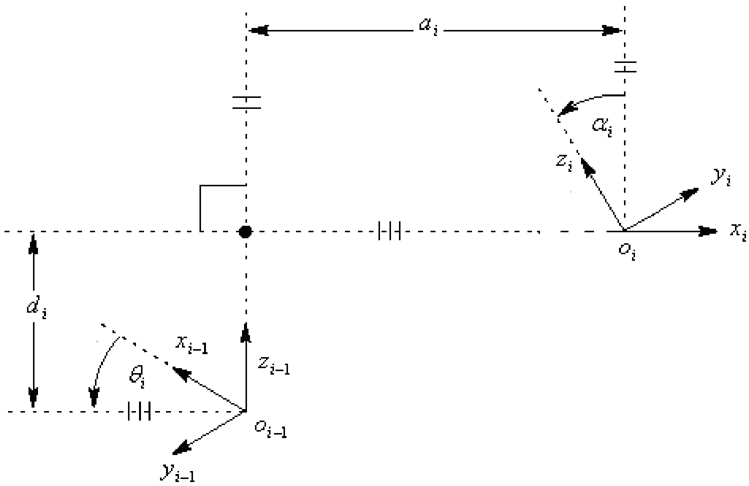

Figure 3: Description of link parameters

2. LINK AND JOINT PARAMETERS

Links are the solid bits between joints. Links have a *proximal* end closest to the base and a *distal* end closest to the tool. The proximal end of the link has the lower joint number. Each type of link has 4 parameters, 2 directions of translation and 2 axes of rotation. These are called the *link parameters*.

Let us consider a binary link of an articulated mechanism as shown in figure 3. It establishes a rigid connection between two successive joints numbered *i* and (*i*+1). Its geometry in terms of size and shape can be described very simply in terms of only two parameters:

1. The distance a_i, measured along the common normal to both axes. The variable a_i is called the link length;
2. The twist angle α_i, defined as the angle between both joint axes. The variable α_i is called the twist angle. The twist angle is measured between the orthogonal projections of joint axes *i* and (*i*+1) onto a plane normal to the common normal.

If the relative motion is restrained to joints of revolute, and prismatic, the relative displacement occurring at joint *i* may also be described in terms of two parameters:

1. The rotation θ_i about the joint axis. The variable θ_i is called joint angle;
2. The displacement d_i along the same axis. The variable d_i is called link offset.

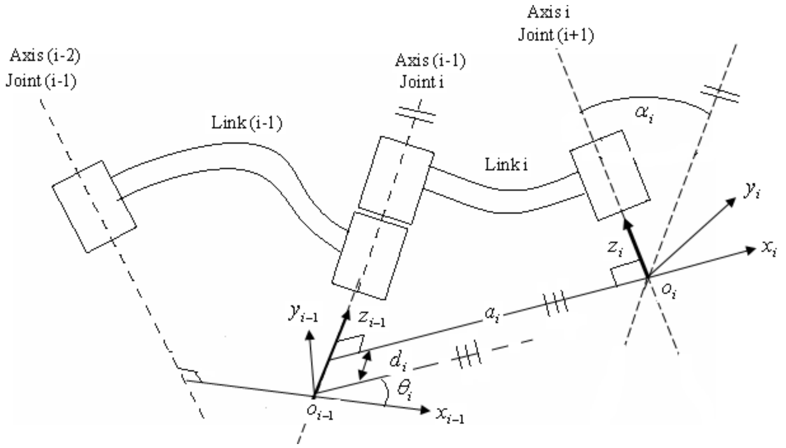

Figure 4: Description of link and joint parameters

2.1 Link Parameters

A link *i* is connected to two other links (i.e. link *i* - 1 and link *i* + 1). Thus two joint axes are established at both ends of the link as shown in figure 4. Joints *i*-1 and *i* are connected by link *i*-1. Joints *i* and *i*+1 are connected by link *i*. The significance of links is that they maintain a fixed configuration between the joints which can be characterized by two parameters a_i and α_i, which determine the structure of the link. They are defined as follows:

1. a_i is the shortest distance measured along x_i axis from the point of intersection of x_i axis with z_{i-1} axis to the origin.
2. α_i is the angle between the joint axes z_{i-1} and z_i axes measured about x_i axis in the right hand sense.

2.2 Joint Parameters

A joint axis establishes the connection between two links. This joint axis will have two normals connected to it, one for each link. The relative position of two such connected links $i-1$ and *i* is given by d_i which is the distance measured along the joint axis z_{i-1} between the common normals. The joint angle θ_i between the common normals is measured in a plane normal to the joint axis z_{i-1}. Hence, the parameters d_i and θ_i are called distance and angle between adjacent links. They determine the relative position of neighboring links. For revolute joint, θ_i varies and d_i is a fixed length (i.e., zero or constant). For prismatic joint, d_i varies and θ_i is zero or constant.

3. LINK FRAMES

Let us define a coordinate frame $o_i x_i y_i z_i$ attached to link i as follows:

1. z - axis is along the rotation direction for revolute joints, along the translation direction for prismatic joints.
2. The z_{i-1} axis lies along the axis of motion of the ith joint.
3. The origin o_i is located at the intersection of joint axis z_i with the common normal to z_i and z_{i-1}.
4. The x_i axis is taken along the common normal and points from joint i to joint i+1.
5. The y_i axis is selected to complete right-hand frame. The y_i axis is defined by the cross product $\boldsymbol{y_i} = \boldsymbol{z_i} \times \boldsymbol{x_i}$.

Showing only z and x axes is sufficient, drawing is made clearer by ***NOT*** showing y axis.

By the above procedure, the link frames for links 1 through n-1 are determined.

4. DENAVIT - HARTENBERG (D-H) CONVENTION

A commonly used convention for selecting frames of reference in robotic application is Denavit-Hartenberg convention. In this convention, the position and orientation of the end-effector is given by

$$H = {}^0T_n = {}^0T_1 \, {}^1T_2 \, {}^2T_3 \ldots {}^{n-1}T_n \qquad (4)$$

where,

$${}^{i-1}T_i = \begin{bmatrix} {}^iR_{i-1} & {}^id_{i-1} \\ 0 & 1 \end{bmatrix} \qquad (5)$$

Many people are not aware that there are two quite different forms of Denavit-Hartenberg representation for the kinematics of serial-link manipulators:

1. Classical convention as per the original paper of Denavit and Hartenberg [1], and used in textbooks such as by Paul [2], Fu et. al [3], or Spong et.al [4].

2. Modified convention as introduced by John J. Craig in his textbook [5] and Tsuneo Yoshikawa in his textbook [6].

Both notations represent a joint as 2 translations (a and d) and 2 angles (α and θ). However the expressions for the link transform matrices are quite different. In short, you must know which kinematic convention your Denavit-Hartenberg parameters conform to. Unfortunately many sources in the literature do not specify this crucial piece of information, perhaps because the authors assume everybody uses the particular convention that they do.

4.1 Classical Convention

The link and joint parameters in the classical convention as shown in figure 5 are as follows:

- Link length, a_i is the offset distance from o_i to the intersection of the z_{i-1} and x_i axes along the x_i axis;
- Twist angle, α_i is the angle from the z_{i-1} axis to the z_i axis about the x_i axis;
- Offset length, d_i is the distance from the origin of the (i-1) frame to the intersection of the z_{i-1} axis with the x_i axis along the z_{i-1} axis;
- Joint angle, θ_i is the angle between the x_{i-1} and x_i axes about the z_{i-1} axis.

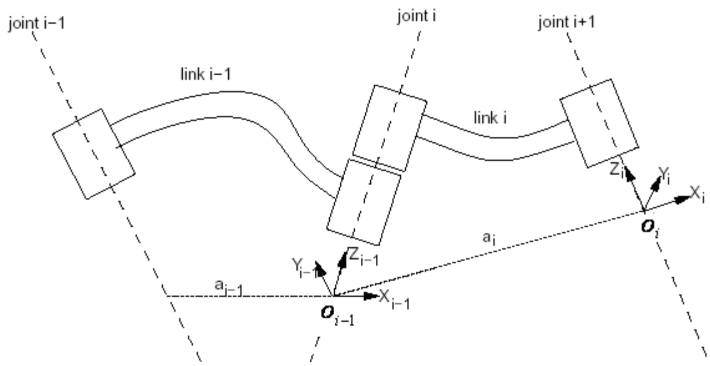

Figure 5: Classical convention

The positive sense of α_i and θ_i is shown in figure 6.

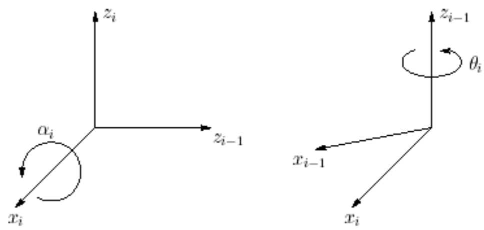

Figure 6: Positive sense of α_i and θ_i

The D-H parameters are tabulated in table 1.

Table 1: D-H parameters for classical convention

Link, i	a_i	α_i	d_i	θ_i
1				
2				

The frame transformation ${}^{i-1}T_i$ describing the finite motion from link i -1 to link i may then be expressed as the following sequence of elementary transformations, starting from link (i – 1):

1. A rotation θ_i about $\mathbf{z}_{i-1}$ axis;
2. A translation d_i along the $\mathbf{z}_{i-1}$ axis ;
3. A translation a_i along the x_i axis
4. A rotation α_i about x_i axis.

The homogeneous transformation ${}^{i-1}T_i$ is represented as a product of four basic transformations as follows:

$$^{i-1}T_i = R(z_{i-1,}\theta_i)T(z_{i-1,}d_i)T(x_{i,}a_i)R(x_i,\alpha_i)$$

=

$$\begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_i & -S\alpha_i & 0 \\ 0 & S\alpha_i & C\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_i & -C\alpha_i S\theta_i & S\alpha_i S\theta_i & a_i C\theta_i \\ S\theta_i & C\alpha_i C\theta_i & -S\alpha_i C\theta_i & a_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (6)$$

Algorithm for classical D-H Convention:

Step - 1: Identify and number the links starting with base and ending with end-effector. The links are numbered from 0 to n. The base frame is {0} and the end-effector frame is {n}. Locate and label the joint axes $z_0, \ldots, z_{n-1}$.

Step - 2: The location of frame to the base is arbitrary. The x_0 axis, which is perpendicular to z_0, is chosen to be parallel to x_1 when the first joint angle variable θ_1 = 0 in the home position. The y_0 axis is defined by the cross product $y_0 = z_0 \times x_0$.

For i = 1, . . . , n, perform Steps 3 to 5.

Step - 3: Locate the origin o_i where the common normal to z_i and z_{i-1} axes intersect at z_i axis. If z_i axis intersects z_{i-1} axis, locate the origin o_i at this point of intersection. If z_i and z_{i-1} axes are parallel, locate the origin o_i in any convenient position along z_i axis.

Step - 4: Establish x_i along the common normal between z_i and z_{i-1} through o_i. The x_i axis is fixed perpendicular to both z_i and z_{i-1} axes and points away from z_i axis. The origin of frame (i) is at the intersection of z_i and x_i axes.

- If the z-axes of two successive joints are intersecting, there is no common normal between them (or it has zero length). We will assign the x-axis along a line perpendicular to the plane formed by the two axes. If z_i and z_{i-1} axes intersect, choose the origin at the point of intersection. The x_i axis will be perpendicular to the plane containing z_i and z_{i-1}. In this case, the parameter a_i equals 0.
- If two joint z-axes are parallel, there are an infinite number of common normals present. We will pick the common normal that is collinear with the common normal of the previous joint. a common method for choosing o_i is to choose the normal that passes through o_{i-1} as the x_i axis; o_i is then the point at which this normal intersects z_i. In this case, d_i is equal to zero. Since the z_i and z_{i-1} axes are parallel, α_i is equal to zero.
- If z_i and z_{i-1} axes coincide, the origin lies on the common axis. If the joint i is revolute, the origin is located to coincide with origin of frame (i) and x_i axis coincides with x_{i-1} axis. If the joint i is prismatic, x_i axis is chosen parallel to x_{i-1} axis and the origin is located at the distal end of the link i.

Step - 5: The y_i axis is selected to complete right-hand frame. The y_i axis is defined by the cross product $y_i = z_i \times x_i$.

Step - 6: Establish the end-effeector frame (o_n) as shown in figure 7. assuming the nth joint is revolute, set $z_n = a$ (approach direction) along the direction z_{n-1} and pointing away from the link n. Establish the origin o_n conveniently along z_n, preferably at the center of the gripper or at the tip of any tool that the manipulator may be carrying. Set $y_n = s$ in the sliding direction along which the fingers the gripper slide to open or close and set $x_n = n$ as $s \times a$. If the tool is not a simple gripper set x_n and y_n conveniently to form a right-hand frame.

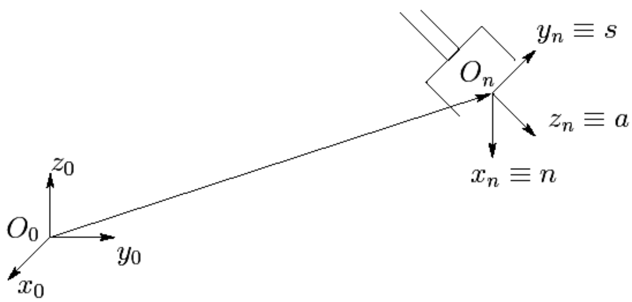

Figure 7: Establishing frame for end-effector

Step - 7: Create a table 2 of link parameters a_i, α_i, d_i, and θ_i.

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Table 2: Link parameters

Link, i	α_i	a_i	d_i	θ_i
1				
2				

- The link length a_i is the shortest distance between z_{i-1} and z_i axes. It is measured as the distance along the direction of x_i from the intersection of z_{i-1} and x_i to the origin of the ith coordinate frame. For intersecting joint axes the value of a_i is zero. It has no meaning for prismatic joints and is set to zero in this case.
- The offset angle, α_i, is measured from z_{i-1} axis to z_i about the x_i axis, again using a right-hand rule. For most commercial manipulators the offset angles are multiples of 90°.
- The distance between links, d_i, is the distance from the x_{i-1} to the x_i axis measured along the z_{i-1} axis. If the joint is prismatic, d_i is the joint variable. In the case of a revolute joint, it is a constant or zero.
- θ_i is the angle from the x_{i-1} to the x_i axis measured about z_{i-1} axis. This is defined using a right-hand rule since both x_{i-1} and x_i *are* perpendicular to z_{i-1}. The direction of rotation is positive if the cross product of x_{i-1} and x_i defines the z_{i-1} axis. θ_i is the joint variable if the joint i is revolute. In the case of a prismatic joint it is a constant or zero.

Step - 8: Form the homogeneous transformation matrices ${}^{i-1}T_i$ by substituting the above parameters into equation (6).

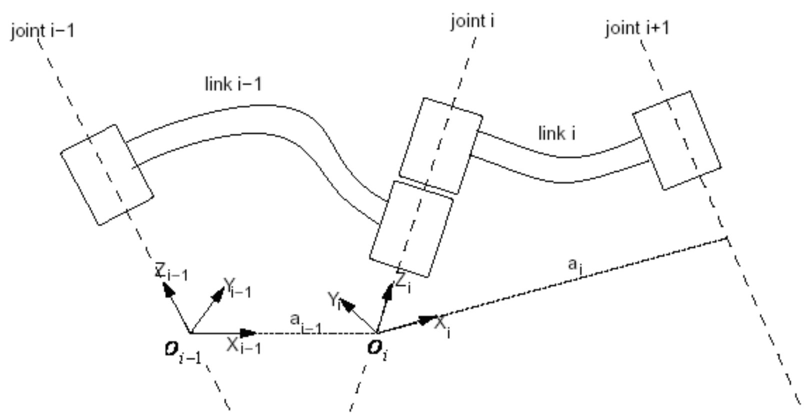

Figure 8: Modified convention

Step - 9: Form ${}^{0}T_n = {}^{0}T_1\,{}^{1}T_2\,{}^{2}T_3 \ldots {}^{n-1}T_n$. This gives the position and orientation of the end-effector frame expressed in the base coordinates.
Note: The origin of the base frame is coincident with the origin of the joint 1. This assumes that the axis the first joint is normal to the *xy* plane.

4.2 Modified Convention

The link and joint parameters in the classical convention as shown in figure 8 are as follows:

- Twist angle, α_{i-1} is the angle between z_{i-1} to z_i measured about x_{i-1}
- Link length, a_{i-1} is the distance from z_{i-1} to z_i measured along x_{i-1}
- Offset length, d_i is the distance from x_{i-1} to x_i measured along z_i
- Joint angle, θ_i is the angle between x_{i-1} to x_i measured about z_i

The D-H parameters are determined as per table 3.

Table 3: D-H parameters for modified convention

Link, i	α_{i-1}	a_{i-1}	d_i	θ_i
1				
2				

The frame transformation ${}^{i-1}T_i$ describing the finite motion from link i -1 to link i may then be expressed as the following sequence of elementary transformations, starting from link (i – 1):

1. A rotation α_{i-1} about x_{i-1}.
2. A translation a_{i-1} along the x_{i-1} axis
3. A rotation θ_i about $\mathbf{z_i}$;
4. A translation d_i along the same axis $\mathbf{z_i}$;

The homogeneous transformation ${}^{i-1}T_i$ is represented as a product of four basic transformations as follows:

$$
{}^{i-1}T_i = R(x_{i-1,}\alpha_{i-1})T(x_{i-1,}a_{i-1})R(z_i,\theta_i)T(z_{i,}d_i)
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha_{i-1} & -S\alpha_{i-1} & 0 \\ 0 & S\alpha_{i-1} & C\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} C\theta_i & -S\theta_i & 0 & 0 \\ S\theta_i & C\theta_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C\theta_i & -S\theta_i & 0 & a_{i-1} \\ S\theta_i C\alpha_{i-1} & C\theta_i C\alpha_{i-1} & -S\alpha_{i-1} & -d_i S\alpha_{i-1} \\ S\theta_i S\alpha_{i-1} & C\theta_i S\alpha_{i-1} & C\alpha_{i-1} & d_i C\alpha_{i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad (7)
$$

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An alternative representation of ${}^{base}T_{end-effector}$ can be written as

$$ {}^{base}T_{end-effector} = {}^{b}T_{e} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & p_x \\ r_{21} & r_{22} & r_{23} & p_y \\ r_{31} & r_{32} & r_{33} & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8) $$

where r_{kj}'s represent the rotational elements of transformation matrix (k and j = 1, 2 and 3). p_x, p_y and p_z denote the elements of the position vector. For a six jointed manipulator, the position and orientation of the end-effector with respect to the base is given by

$$ {}^{0}T_6 = {}^{0}T_1(q_1)\,{}^{1}T_2(q_2)\,{}^{2}T_3(q_3)\,{}^{3}T_4(q_4)\,{}^{4}T_5(q_{51})\,{}^{5}T_6(q_6) \quad (9) $$

where q_i is the joint variable (revolute or prismatic joint) for joint i, (i = 1, 2, …6).

Algorithm for modified D-H Convention:

Step - 1: Assigning of base frame: the base frame {0} is assigned to link 0. The base frame {0} is arbitrary. For simplicity chose z_0 along z_1 axis when the first joint variable is zero. Using this convention, we have $a_0 = 0$ and $\alpha_0 = 0$. This also ensures that $d_1 = 0$ if the joint is revolute and $\theta_1 = 0$ if the joint is prismatic.

Step - 2: Identify links. The link frames are named by number according to the link to which they are attached (i.e. frame {i} is attached rigidly to link *i*). For example, the frame {2} is attached to link 2.

Identify joints. The z-axis of frame {i}, called z_i, is coincident with the joint axis *i*. The link *i* has two joint axes, z_i and z_{i+1}. The z_i axis is assigned to joint *i* and z_{i+1} is assigned to joint (*i*+1).

For *i* = 1,….., *n* perform steps 3 to 6.

Step – 3: Identify the common normal between z_i and z_{i+1} axes, or point of intersection. The origin of frame {i} is located where the common normal (a_i) meets the z_i axis.

Step – 4: Assign the z_i axis pointing along the ith joint axis.

Step – 5: Assign x_i axis pointing along the common normal (a_i) in the direction from z_i axis to z_{i+1} axis. In the case of a_i = 0, x_i is normal to the plane of z_i and z_{i+1} axes.

- As seen from figure 3.7, the joints may not necessarily be parallel or intersecting. As a result, the *z*-axes are skew lines. There is always one line mutually perpendicular to any two skew lines, called the common normal, which has the shortest distance between them. We always assign the *x*-axis of the local reference frames in the direction of the common normal. Thus, if a_i represents the common normal between z_i and z_{i+1}, the direction x_i is along a_i.

- If two joint *z*-axes are parallel, there are an infinite number of common normals present. We will pick the common normal that is collinear with the common normal of the previous joint.
- If the *z*-axes of two successive joints are intersecting, there is no common normal between them (or it has zero length). We will assign the *x*-axis along a line perpendicular to the plane formed by the two axes.

Step – 6: The y_i axis is selected to complete right-hand coordinate system.

Step – 7: Assigning of end-effector frame: If the joint *n* is revolute, the direction of x_n is chosen along the direction of x_{n-1} when $\theta_n = 0$ and the origin of frame {n} is chosen so that $d_n = 0$. If the joint *n* is prismatic, the direction of x_n is chosen so that $\theta_n = 0$ and the origin of frame {n} is chosen at the intersection of x_{n-1} with z_n so that $d_n = 0$.

Step – 8: The link parameters are determined as mentioned in table 4.

Table 4: Link parameters

Link, *i*	a_{i-1}	α_{i-1}	d_i	θ_i
1				
2				

- a_{i-1} = the distance from z_{i-1} to z_i measured along x_{i-1}
- α_{i-1} = the angle between z_{i-1} to z_i measured about x_{i-1}
- d_i is the distance from x_{i-1} to x_i measured along z_i
- θ_i is the angle between x_{i-1} to x_i measured about z_i

Step - 9: Form ${}^0T_n = {}^0T_1\,{}^1T_2\,{}^2T_3 \ldots {}^{n-1}T_n$. This gives the position and orientation of the end-effector frame expressed in the base coordinates.

5. CASE STUDIES

RR planar manipulator is used to differentiate between D-H classical convention and D-H modified convention.

5.1 Case Study for D-H Classical Convention

Formulate the forward kinematic model of two-degree of freedom RR planar manipulator as shown in figure 9. Find the home position of the manipulator. Use classical D-H convention.

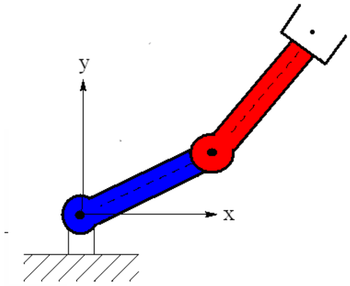

Figure 9: Two-degree of freedom RR Planar manipulator

Solution:
The manipulator consists of two joints. The two joints are revolute joints. The scheme of frame assignment of the manipulator is shown in figure 10. For revolute joint, d = 0.

The manipulator consists of two joints (i.e. n = 2). The axis of revolute joint is perpendicular to the paper (figure 10).

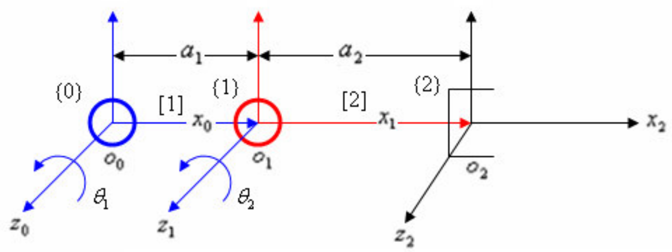

Figure 10: Frame assignment for two-degrees of freedom RR planar manipulator

Step -1: The joints are revolute type. The two links are numbered [1] and [2]. The base frame is {0} and frames for the rest of links are numbered {1} and {2}. The joint axes are labeled as z_0, z_1, and z_2.

Step -2: The location of frame to the base is arbitrary. The x_0 axis, which is perpendicular to z_0, is chosen to be parallel to x_1 when the first joint angle variable θ_1 = 0 in the home position. The y_0 axis is defined by the cross product $y_0 = z_0 \times x_0$.

For i = 1, perform steps 3 to 5.

Step -3: The link 1 has two joint axes, z_0 and z_1. The z_0 axis is assigned to joint 1. The z_1 axis is assigned to joint 2. There is a common normal between z_0 and z_1 axes. Axis z_1 is parallel to z_0 axis. The origin o_1 is located in any convenient position along z_1 axis as shown in figure 3.12.

Step -4: Axis x_1 is established along the common normal between z_0 and z_1 through 0_1.

Step -5: The y_1 axis is defined by the cross product of $y_1 = z_1 \times x_1$.

For $i = 2$

Step -3: The link 2 has one joint axis, z_1 which is common to link 1 and link 2. The second end of link 2 is rigidly connected to the end-effector. The z_2 axis is set parallel to z_1 axis. Since z_1 and z_2 axes are parallel, the origin o_2 is located in any convenient position along z_2 axis as shown in figure 3.12.

Step -4: The common normal between z_1 and z_2 axes is x_2. Since the joint 2 is revolute, x_2 axis is chosen in the direction parallel to x_1 axis and passing through the origin o_2.

Step -5: The y_2 axis is defined by the cross product of $y_2 = z_2 \times x_2$.

Step -6: Establish the end-effector frame {2} as shown in figure 3.12.

Step -7: The joint-link parameters are tabulated in table 5.

Table 5: Joint-link parameters of classical convention

Link, i	a_{i-1}	a_{i-1}	d_i	θ_i
1	a_1	0	0	θ_1
2	a_1	0	0	θ_2

Step -8: Form the homogeneous transformation matrices ${}^{i-1}T_i$ by substituting the above parameters into equation (3.7)

$$ {}^{0}T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{1}T_2 = \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Step- 9: Form ${}^0T_2 = {}^0T_1\,{}^1T_2$. This gives the position and orientation of the tool frame expressed in base coordinates.

$$ {}^0T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The home potion of the manipulator is corresponding $\theta_1 = \theta_2 = 0$. By substituting these values in ${}^0T_2$, we get the home position of the manipulator.

$$ {}^0T_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 + a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

5.2 Case Study for D-H Modified Convention

Derive forward kinematics of two-degree of freedom RR planar manipulator as shown in figure 11. Find the home position of the manipulator. Use modified D-H convention.

Solution:

The manipulator consists of two joints. The two joints are revolute joints. The scheme of frame assignment of the manipulator is shown in figure 11. For revolute joint, *d* = 0. The axis of revolute joint is perpendicular to the paper.

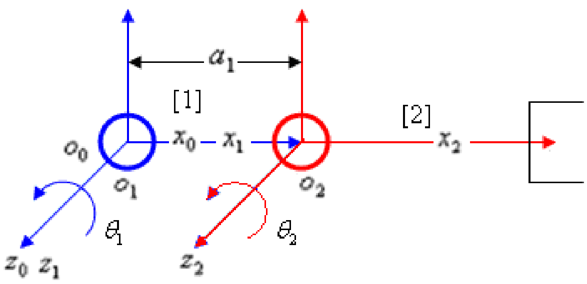

Figure 11: Frame assignment for two-degrees of freedom RR planar manipulator

Step -1: The base frame {0} is assigned to link 0. The base frame {0} is arbitrary. For simplicity chose z_0 along z_1 axis when the first joint variable is zero. Using this convention, we have $a_0 = 0$ and $\alpha_0 = 0$. This also ensures that $d_1 = 0$ as the joint 1 is revolute.

Step - 2: The manipulator consists of two links. The two links are numbered [1] and [2]. The link frames are numbered as {1} and

{2}. The joint 1 is between link 0 and link 1 and its z axis is z_1. The joint 2 is between link 1 and link 2 and its z axis is z_2.
For i = 1, perform steps 3 to 6.

Step – 3: There is a common normal between z_1 and z_2 axes. The origin o_1 of frame {1} is located where the common normal (a_1) meets the z_1 axis.

Step – 4: Assign the z_1 axis pointing along the 1st joint axis.

Step – 5: The x_1 axis is pointing along the common normal (a_1) in the direction from z_1 axis to z_2 axis and passing through the origin o_1.

Step – 6: The y_1 axis is selected to complete right-hand coordinate system.
For i = 2, perform steps 3 to 6.

Step – 3: The link 2 consists of only one joint that is z_2 axis and the other end of link 2 is rigidly fixed to the end-effector. There is a common normal between z_2 and z_3 axes (here: the z_3 axis is belonging to the end-effector). The origin o_2 of frame {2} is located where the common normal (a_2) meets the z_2 axis.

Step – 4: Assign the z_2 axis pointing along the 2nd joint axis.

Step – 5: The x_2 axis is pointing along the common normal (a_2) in the direction from z_2 axis to z_3 axis and passing through the origin o_2

Step – 6: The y_2 axis is selected to complete right-hand coordinate system.

Step – 7: Assigning of end-effector frame: The direction of x_3 aligns with x_2 when $\theta_3 = 0$ and the origin of frame {3} is chosen so that $d_3 = 0$.

Step – 8: The link parameters are determined as mentioned in table 6.

Table 6: Link parameters of modified convention

Link, i	a_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	a_1	0	0	θ_2

Step - 9: Form ${}^0T_2 = {}^0T_1\,{}^1T_2{}^{2}$. This gives the position and orientation of the end-effector frame expressed in the base coordinates.

$$
{}^0T_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$^{1}T_{2} = \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{0}T_{2} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_{12} \\ s_{12} & c_{12} & 0 & a_1s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Note: The forward kinematics ends at a frame, whose origin lies on the last joint axis z_2, therefore, a_2 doest not appear in the link parameters.

5.3 Difference between Classical and Modified Conventions

The position and orientation of the end-effector frame expressed in the base coordinates obtained by the modified D-H convention are given by:

$$^{0}T_{2} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_1c_2 \\ s_2 & c_2 & 0 & a_1s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_{12} \\ s_{12} & c_{12} & 0 & a_1s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematics ends at a frame, whose origin lies on the last joint axis z_2 as per modified D-H convention, therefore, a_2 doest not appear in the link parameters as shown in figure 12. The home potion of the manipulator is corresponding $\theta_1 = \theta_2 = 0$. By substituting these values in $^{0}T_{2}$, we get the home position of the manipulator.

$$^{0}T_{2} = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

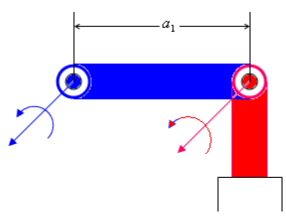

Figure 12: Home position of RR planar manipulator as obtained by modified D-H convention.

The position and orientation of the tool frame derived from the classical D-H convention are given by

$$ {}^{0}T_2 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1c_1 \\ s_1 & c_1 & 0 & a_1s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_2 & -s_2 & 0 & a_2c_2 \\ s_2 & c_2 & 0 & a_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_{12} & -s_{12} & 0 & a_1c_1 + a_2c_{12} \\ s_{12} & c_{12} & 0 & a_1s_1 + a_2s_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

The home potion of the manipulator is corresponding $\theta_1 = \theta_2 = 0$. By substituting these values in ${}^{0}T_2$, we get the home position of the manipulator as shown in figure 13.

$$ {}^{0}T_2 = \begin{bmatrix} 1 & 0 & 0 & a_1 + a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

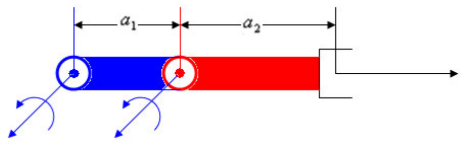

Figure 13: Home position of RR planar manipulator as obtained by classical convention.

6. CONCLUSIONS

Two conventions have been established for assigning coordinate frames, each of which allows some freedom in the actual coordinate frame attachment.

D-H parameter	Classical convention	Modified convention
Joint axis	z_{i-1} is for joint i	z_i is for joint i
Link length (a_i)	The distance from o_i to the intersection of the z_{i-1} and x_i axes along the x_i axis	The distance from z_i to z_{i+1} measured along x_i
Twist angle(α_i)	The angle from the z_{i-1} axis to the z_i axis about the x_i axis	The angle between z_i to z_{i+1} measured about x_i
Offset length (d_i)	The distance from the origin of the (i-1) frame to the intersection of the z_{i-1} axis with the x_i axis along the z_{i-1} axis	The distance from x_{i-1} to x_i measured along z_i
Joint angle (θ_i)	The angle between the x_{i-1} and x_i axes about the z_{i-1} axis	The angle between x_{i-1} to x_i measured about z_i

This paper discusses both the conventions (classical and modified) for the forward kinematics of RR manipulator. One can have choice of using any one method. Most of the universities are having several affiliated technical institutions. In such situations, the question paper is set by the university for the end examinations. It may happen that the answer scripts may consist of either of the conventions. Teachers are requested to correct the answer scripts as per the convention followed by the student but not the convention that he taught in the class room.

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