**Application of Eulerian and Lagrangian Couplings to Estimate the Influence of Shock Pressure Loading on the Titanium Submersible hull Using Finite Element Analysis**



# **Engineering**

**KEYWORDS :** Titanium, submersible hull, shock pressure load, finite element analysis, Eulerian-Lagrangian coupling.

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**ABSTRACT** The aim was to estimate the influence of shock pressure loading on the submersible hull using finite element analy*sis. The fluid medium was molded based on Tait's equation of state. The equation of state from Jones-Wilkins-Lee (JWL) was used to describe the detonation products of explosives. The explosion and fluid were interfaced using Eulerian-Eulerian coupling and the fluid and shell were interfaced using arbitrary Lagrangian-Eulerian coupling. The plastic displacement of the submerged hull has been found to be 0.0412 m for explosion charge of 15 kg. The titanium submersible fails at the shock load factor of 0.45 for the explosion charge of 25 kg.* 

### **INTRODUCTION**

Underwater explosions are very important and complex problems for naval surface ships or submarines, since detonations near a ship can damage the vessel. The inelastic behavior of structures to dynamic loads such as impulse, blast and underwater shock is of great importance in many fields such as marine and ocean industries. The problem is fairly complex involving material and geometric non-liniearities. Huang and Kiddy (1995) studied the transient interaction of a spherical shell with an underwater explosion shock wave and subsequent pulsating bubble, based on their approach on the finite element method coupled with the Eulerian–Lagrangian method. According to their results, the structural response, as well as interactions among the initial shock wave, the structure, its surrounding media and the explosion bubble must be considered. Kwon and Fox (1993) applied numerical and experimental techniques to investigate the nonlinear dynamic response of a cylinder subjected to a side-on, far-field underwater explosion. Comparisons between the strain gage measurements and the numerical results at different locations revealed a good agreement. Shin and Chisum (1997) employed a coupled Lagrangian–Eulerian finite element analysis technique as a basis to investigate the response of an infinite cylindrical and a spherical shell subjected to a plane acoustic step wave.

When a submerged structure subjected to underwater explosion loading, it is important to predict the structural response to the shock wave. Furthermore, in the case of the explosion occurring close to the structure, a high velocity water jet penetrating the gas bubble occurs. This water jet is extremely efficient in producing damage.

The purpose of this paper was to demonstrate the application of Eulerian-Eulerian coupling to interface the explosion and fluid medium and Lagrangian-Eulerian coupling to interface the fluid and submersible hull. The objective was also to estimate the influence of shock pressure loading on the submersible hull using finite element analysis.

## **THEORETICAL BACKGROUND**

The sequence of underwater explosion and finite element modeling are discussed.

## **Shock Wave Pressure:**

The underwater shock wave generated by the explosion is superimposed on the hydrostatic pressure. The pressure history  $P(t)$  of the shock wave at a fixed location starts with an instantaneous pressure increase to a peak  $P_{max}$  followed by a decline which initially is usually approximated by an exponential function. Thus, according to the empirical equation of Cole (1948):

$$
P(t) = P_o e^{-t/\theta} 0 \ll t \ll \theta \tag{1}
$$

The peak pressure  $(P_{_{o}})$  and the decay constant  $(θ)$  are given by

$$
P_o = 52.16 \times 10^6 \left( W^{1/3} / R \right)^{1.13} \tag{2}
$$

$$
\theta = 92.5 \times W^{1/3} (W^{1/3}/R)^{-0.22}
$$
 (3)

where *W* is the charge weight (kg) and *R* is the stand-off distance (m).

Because of the spherical spreading nature of the shock wave, the wave reaches different locations at different times, i.e. there is time delay. The time delay  $(t_a)$  can be calculated using the radial distance at any location  $(R)$ , the shortest radial distance  $(R_0)$  and the saund wave velocity  $(c)$  as follows: the sound wave velocity (*c*), as follows:

$$
t_d = (R - R_0) / c \tag{4}
$$

 $\overline{\phantom{a}}$ <br>By incorporating the time delay, Eq. (1) is rewritten in the following form: By incorporating the time delay, Eq. (1) is rewritten

$$
P(t) = P_o e^{-(t - t_d)/\theta} 0 \ll t \ll \theta \tag{5}
$$

## **Shock Wave Velocity:**

As the wave travels from the explosion, the profile of the shock wave broadens and the amplitude reduces as shown in Fig-1. The velocity in the vicinity of the explosion depends on the peak pressure of the shock wave and the acoustic velocity, as given by

$$
c_s = c_a \times (1 + 6 \times 10^{-10} P_o)
$$
 (6)  
As the shock wave propagates, it sets the water  
particle in the vicinity in motion. The water particle  
velocity associated with the shock wave is given by  
 $v(t) = P(t)/\rho c$  (7)

where  $\rho$  is the density of the fluid medium.



Fig-1. Pressure waves and bubble phenomena of underwater explosion.

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 $(16)$ 

# intervater explosion.  $u$ ludel water explosion.

The initial high pressure in the product gases of explosion decreases considerably after the primary shock pulse is emitted. The inside pressure in the gas bubble is much higher than the surrounding hydrostatic pressure, which tends to push the water and expand the gas bubble. At this stage, the gas bubble expands rapidly and consequently the pressure inside the bubble decreases and the kinetic energy accelerates the surrounding water till the bubble expands to the maximum. As the gas bubble expands to the maximum radius, the gas pressure falls below the hydrostatic pressure, the contraction of gas bubble starts and continues until the pressure inside becomes insignificant. Hence, the gas bubble undergoes repeated cycles of expansion and contraction. Ine initial high pressure in the product gases

The maximum radius  $(R_{max})$  during the first pulsation and the duration (*T*) of the first pulsation are given by  $R_{max} = 3.3 \times (W/Z_0)^{1/3}$  (8)

 $T = 2.06 \times (W^{1/3}/Z_0^{5/6})$  (9)  $Z_o = D + 10$  (10)

where, *D* is water depth,  $Z_o$  is the reference depth.

## **Secondary Shock Wave:**

During the contraction phase of the gas bubble oscillation, when the bubble reaches its minimum, a pressure pulse known as the secondary shock wave, of small amplitude is emitted. The peak pressure of the secondary pressure pulse is given by  $P_2 = 2590 \times (W^{1/3}/R)$  (11)

## **Gas Bubble Migration:**

When the gas bubble has last buoyancy, the migration of gas bubble occurs. The migration of the gas bubble from the location of the explosive charge up to the location corresponding to the first bubble pulse is given by

$$
m = (90/Z_o)W^{1/2} \tag{12}
$$

# **Hull Shock Factor:**

Since a ship can be subjected to a large variety of underwater explosion (variation in charge weight, standoff distance), the relation between attack severity and geometry must be determined. For damage predictions for submarines, this factor is referred to as the Hull Shock Factor (HSF). It has been found that

$$
HSF = \sqrt{W/R} \tag{13}
$$

where, *W* is the charge weight and *R* is the standoff distance.

# **Finite Element Modeling:**

For a fully or partially submerged structure subjected to an underwater shock wave, the structure may exhibit material and geometrical nonlinear behavior. Based on the theorem of virtual displacement, the

governing equation of the problem can be expressed in matrix form (Chennakesava R Alavala, 2008) as  $B_1$  and the theorem of virtual displacement, the theorem of virtual displacement, the set of virtual displacement,  $\frac{1}{2}$ 

Based on the theorem of virtual displacement, the

$$
[Ms]\{\ddot{u}\} + [Cs]\{\dot{u}\} + [Ks]\{u\} = \{f\}
$$
 (14)  
where,

 $[M_s] = \int \rho_s [N]^T [N] dv, [C_s] = \int \rho_s \alpha_c [N]^T [N] dv,$  $[K_{s}] = \int [B]^{T} [D] [B] dv$  and  $\{f\} = \int [N]^{T} f dv$ 

 $[N<sub>s</sub>]$ ,  $[C<sub>s</sub>]$  and  $[K<sub>s</sub>]$  are the structural mass, damping and stiffness matrices, respectively.  $[N]$ ,  $[B]$  and  $[D]$ and strinkes matrices, respectively.  $\left[1, 1\right]$ ,  $\left[2, 1\right]$  and  $\left[2, 1\right]$  $\frac{1}{2}$  and  $\frac{1}{2}$  are the structure,  $\frac{1}{2}$  and  $\frac{1}{2}$   ${f}$  are the structural displacement and the external force vector, respectively.  $\ldots$  and  $\rho$  takes the plastic tangent stiffness.

For a structure submerged in an infinite acoustic medium, the governing equation of the wet surface of Approximation as given below:  $\mathbf{r}$  medium, the governing equation of the wet surface of the wet the shell is based on the Doubly Asymptotic

$$
[M_f]\{\ddot{P}_s\} + \rho_f c[A_f]\{\dot{P}_s\} + \rho_f c[\varphi_f][A_f]\{P_f\} =
$$
  
\n
$$
\rho_f c[M_f]\{\dot{v}_s\} + [\varphi_f][M_f]\{v_s\}
$$
 (15)

 $[\varphi_{\epsilon}] =$ 

$$
\alpha \rho_f c [A_f] [M_f]^{-1}
$$

 $[M_f]$  is the symmetric fluid mass matrix  $\alpha$  is the scale parameter bounded  $0 \ll \alpha \ll 1$ ,  $\rho_f$  and *c* are the fluid density and sound velocity, respectively.  $\{v_s\}$  is the vector of scattered-wave fluid particle velocities normal to the structural surface.

The fluid surface is coupled to the structural response by the following equation

$$
\{v_s\} = [G]^T \{ \dot{u} \} - \{ v_i \}
$$
  
where  $\{ v_i \}$  is the fluid incident velocity. (17)

# **MATERIALS AND METHODS**

The physical model of a submersible hull is shown Fig-2. The major dimensions of the submersible hull are as follows:

Diameter = 3 m Length = 9.5 m Thickness = 0.035 m



Fig-2. Physical model of submersible hull

The finite element analysis of the submersible hull was carried out using DYTRAN non-linear finite element code. For the finite element analysis, the

explosion, fluid and submersible hull were modeled and submersible hull were modeled and submersible hull were

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normal line passing through the centerline of the submersible hull at a distance of 5 m. The exploded charge weight was varied from 1kg to 25 kg. explosion, fluid and submersible hull were modeled exprosion, nuite and submersible nun were inducted<br>as an integral unit. The fluid and explosion were as an integral unit. The fluid and explosion were<br>meshed with 8 node Eulerian solid element (Fig-3a). meshed with 6 node Euterian sond element (Fig-5a).<br>The number of elements was 106160. The fluid domain of 5 m width in transverse direction and 3 m submant of  $\beta$  in width in that system extends and  $\beta$  in width in the longitudinal direction from the with in the longitudinal direction from the submersible hull was considered for modeling. The submersible hull was considered for modering. The submersible hull was discretized with 4 node Enderstone han was discretized with 4 hode Eagrangian Cicincin (11g-50). The number of elements was 7708. The explosion and fluid were interfaced using an Eulerian-Eulerian coupling. The Interfaced using an Eulerian-Eulerian coupling. The fluid and the submersible hull were interfaced using Initia and the submersione han were interfaced using<br>Lagrangian-Eulerian coupling. In the present investigation, the explosion was assumed on the

element code. For the finite element analysis, the



Fig-3. Finite element modeling (a) fluid domain and explosion and (b) submersible hull

For the finite element analysis, the fluid was modeled using Eulerian solid element with Tait's equation of state and the explosion was modeled using Eulerian solid element with JWL equation of state. The material constants used in the Tait's equation of state are as follows:

> $\rho = 1025 \ kg/m^3$  $a = 48402.7105 Pa$  $b = 3.01E8$  Pa  $R = 7.15$  $C_0 = 1450 \ m/s$

The equation of state from Jones-Wilkins-Lee (JWL) is used to describe the detonation products of explosives.

$$
p = A \left( 1 - \frac{\omega}{R_1 V} \right) \exp(-R_1. V) + B \left( 1 - \frac{\omega}{R_2 V} \right) \exp(-R_{12}. V) + \frac{\omega e_0}{V}
$$
\n(18)

R1 = 4.15

The ratio  $V = \rho_e / \rho$  is defined by using  $\rho e$  is the density of explosive (solid part) and  $\rho$  is the density of detonation products. The parameters A, B, R1,  $R2$  and  $\omega$  are given below:

$$
\rho = 1610 \text{ kg/m}^3
$$
  
\nA = 371.2 GPa  
\nB = 3.2306 GPa  
\nR<sub>1</sub> = 4.15  
\nR<sub>2</sub> = 0.95  
\n $\omega = 0.3$ 

The submergible hull was made of elastic Ti-6Al-4V alloy. The material properties are given below:  $T = 112.9 \text{ CPa}$ 

 $E = 113.8$  GPa  $v = 0.342$  $\sigma_{uts} = 950 \text{ MPa}$  $\sigma_{\text{ys}} = 880 \text{ MPa}$  $\overline{\phantom{a}}$ σ<sub>cs</sub> = 970 MPa<br> $G = 44 GPa$  $\tau = 550 MPa$  $\rho = 4430 kg/m^3$ 

The analysis of the coupled field problem was solved using explicit integration scheme. Three incremental time steps of 0.05, 0.1 and 0.2 microseconds were used for the analysis. The explosion and fluid were interfaced using Eulerian-Eulerian coupling and the fluid and shell were interfaced using arbitrary Lagrangian-Eulerian coupling. The initial conditions used in the explosion were specific internal energy  $(4.16 \times 10^6 \text{ K/kg})$  and detonation velocity (6730 m/s). The explosive element was modeled using eight node Eulerian solid elements. The element length was 0.426 m. The stand-off distance was 5 m.

#### **RESULTS AND DISCUSSION**

The shock pressure loading of the fluid medium was applied on the surface of on the submersible hull through arbitrary Lagrangian-Eulerian coupling. The peak pressures were calculated for all the cases are given in table 1. The peak value was 25 MPa for a charge weight of 15 kg.







**Fig-4. Displacement-time history for charge weight of 20 kg**

#### **Displacement-Time History:**

The displacement-time history of the submergible hull exposed to the explosion for a charge weight of 15 kg is shown in Fig-4. The deformation of the submerged hull was obtained from the maximum displacement by subtracting the elastic deformation. The maximum displacement was 0.0412 m at 9.605 milliseconds. The displacement contours are shown in Fig-5. The plastic displacement was found to be high on the submergible hull side exposed to the explosion (Fig-5a). The elastic displacement was found on the opposite of the explosion (Fig-5b). There is no permanent displacement on the back side.



**Fig-5. Displacement of submerged hull at HSF = 0.34 (a) surface exposed to explosion and (b) Surface on the rear side of submersible hull**

### **Stress-Time History:**

The tensile stress contours of the submergible hull subjected to for the explosion charge weight of 15 kg is shown in Fig-6. The maximum tensile stress along the longitudinal direction of the submergible hull is 474 MPa (Fig-6a) while the maximum tensile stress along the circumferential direction of the submergible hull is 491 MPa (Fig-6b). The maximum allowable tensile stress is 880 MPa. The tensile stresses are less than the allowable stress. This indicates that the hull does not fail. The maximum shear stress is 230 MPa in the submergible hull (Fig-7). Comparing this value with the allowable shear stress (550 MPa), the fail does not occur in shear also.

#### **Impact of Shock Wave:**

The deformation increases with the increase of shock load as shown in Fig-8. The stress induced increases with the increase of shock load along the longitudinal and circumferential directions as shown in Fig-9. When the hull shock factor exceeds 0.40 the submersible hull fails.



**Fig-6. Tensile stress of submerged hull at SF = 0.34 (a) longitudinal and (b) circumferential**



**Fig-7. Effective shear stress at SF = 0.34** 







#### **Fig-9. Effect of shock load on tensile stress of submerged hull (a) longitudinal and (b) circumferential**

#### **CONCLUSIONS**

The plastic displacement of the submerged hull has been found to be 0.0412 m for explosion charge of 15 kg. The titanium submersible fails at the shock load factor of 0.45 for the explosion charge of 25 kg. The damage has occurred in the submersible hull surface exposed to explosion only.

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