

Computational Fluid Dynamic Analysis of Flow Over a Sine Wave-Shaped Wall

This paper consolidates the use of computational fluid dynamics for the analysis of flow over a sine wave shaped wall. In this approach the linear elliptic differential was reduced to laplacian with appropriate gradient boundary conditions. The geometry was meshed with straight-sided triangular elements. The analytical and the software package results were compared. The maximum error was located at the midspan of the wall.

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Introduction

A vital problem in fluid mechanics, directly obtaining the accurate analysis using the finite element concepts, is

dimensional aerodynamic flow in a cascade using triangular elements. Two-dimensional aerodynamic configuration solutions are given by Miessner (1973) and Sarpakaya and

Hiriyar (1975) conclude results for axisymmetric thrust reversers involving a free surface

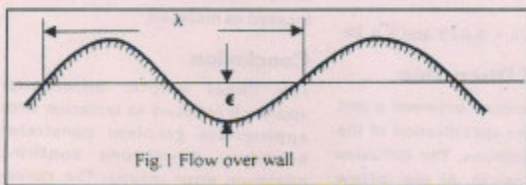


Fig. 1 Flow over wall

potential flow. For the steady irrotational flow of an incompressible inviscid fluid, the governing eulerian equations can be reduced to the laplacian written on either of two velocity potential functions. This yields an equivalent for the steady subsonic compressible flow about slender aerodynamic bodies.

DeVries and Norie (1971) obtained a finite element solution for two-

of unknown location.

Potential flow

In the eulerian description for fluid mechanics [4], the fundamental laws become expressed for a region in space through which mass is converted by the local velocity vector u . From Reynolds transport theorem, the equivalent differential statement for conservation of mass of an incompressible fluid is vanishing

divergence of the velocity field.

$$L(u) = \nabla \cdot u = 0 \quad \text{---(1)}$$

For a potential flow field, the velocity field is irrotational; therefore, the curl of the velocity vanishes

$$\text{Curl } u = \nabla \times u = 0 \quad \text{---(2)}$$

From vector field theory, the equation (2) is identically satisfied by definition of the scalar potential flow Φ as

$$u = -\nabla \Phi \quad \text{---(3)}$$

Where the negative sign is by convention.

Substitution of the equation (3) into (1) yields the differential equation governing potential flow:

$$L(\Phi) = \nabla^2 \Phi = 0 \quad \text{---(4)}$$

Where ∇^2 is the laplacian.

The boundary conditions for the equation (4) are provided by the equation (3).

In the two-dimensions, for incompressible flows, the potential function is called stream function (C). The differential equation governing stream function is

$$L(C) = \nabla^2 C = 0 \quad \text{---(5)}$$

The equations (4) and (5) are linear elliptic partial differential equations.

The finite element solution is to solve a linear elliptic partial differential equation with appropriate gradient constraint boundary conditions. The finite element approximation is

$$E(\Phi^h, \Phi^h) = \frac{1}{2} \sum_{T \in \mathcal{T}_h} \int_T \nabla \Phi^h \cdot \nabla \Phi^h \, dz \quad \text{---(6)}$$

3. Formulation

The flow over a sine wave-shaped wall is shown in fig.1. The geometry was meshed with straight-sided triangular elements. The boundary conditions for scalar and vector potentials are shown in fig.2. The boundary conditions are:

$$\nabla\phi \cdot \hat{n} = 0 \quad (7)$$

where \hat{n} is the local outward pointing unit normal vector.

For small aspect ratio,

$\frac{t}{\lambda} < 1$, the flow tangency wall boundary condition (7) can be linearised. For the sine wave-shaped wall,

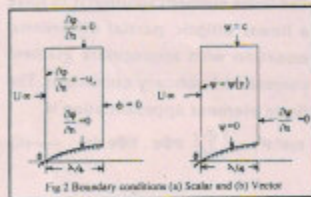
$$Y_w(x) = \varepsilon \sin \frac{2\pi x}{\lambda} \quad (8)$$

The infinite series solution degenerates to one nonzero term is

$$\phi(x, y) = u_\infty \left[-\varepsilon + \varepsilon \cos \left(\frac{2\pi x}{\lambda} \right) \exp \left(-\frac{2\pi y}{\lambda} \right) \right] \quad (9)$$

The assumptions made in the finite element formulation are:

- The flow is incompressible
- The element thickness is uniform
- No mass flux across the element face
- The difference between scalar potential and vector potential is the specification boundary only
- $Y = \lambda$ and $x = \lambda/4$.
- The wall is impervious



The computational fluid flow analysis was carried out using NISA software package.

240 triangular elements were used to

mesh the geometry. The mesh generation is shown

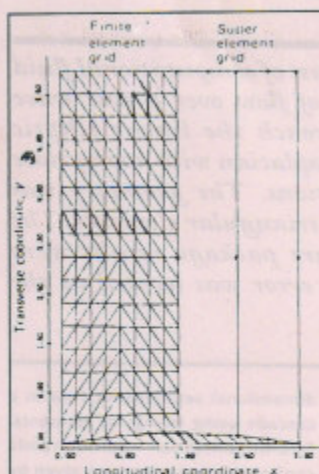


Fig.3 Wave shaped wall discretization

in fig.3.

$$U_\infty = 30 \text{ m/s}, \quad \varepsilon/\lambda = 0.025 \text{ and } \lambda = 2\pi$$

Results and Discussion

The main difference between ϕ and ψ solution is the specification of the boundary conditions. The diffusion matrix is identical. At the inflow boundary $x = 0$, the imposed free stream velocity U_∞ becomes a gradient boundary condition on ϕ . The normal gradient of ϕ vanishes at both the top and bottom of the domain, since $Y = \lambda$. This is a good approximation to streamline, as the wall is impervious. The downstream boundary is defined an $x = \lambda/4$ as the geometry is symmetric in nature. The scalar potential is constant on the downstream symmetry plane.

The equipotential distribution is shown in fig.4. It can be observed that there is normal intersection of the equipotentials with the wall and the upper free stream boundary. The

analytical and the software package results are compared. The average is

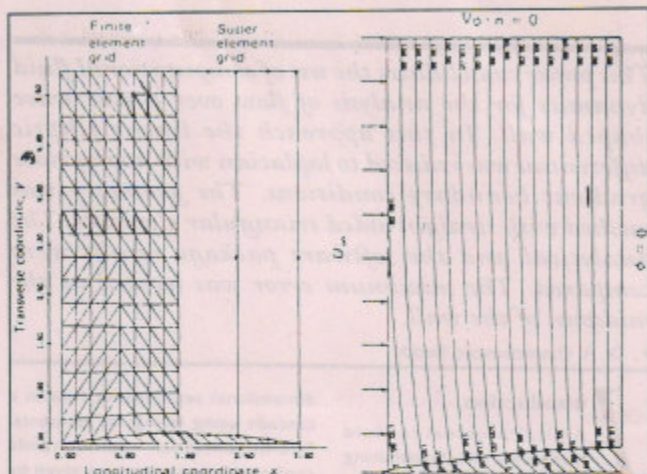


Fig.4 Computed equipotential distribution

about 1.5%. The maximum error is located at midspan.

Conclusion

The linear elliptic differential approach reduced to laplacian with appropriate gradient constraint boundary conditions confirms minimum error results. The curved-sided triangular elements can yield error-free results.

References

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