STUDIES ON THE GAS PRESSURE FORMED Pb-Sn EUTECTIC ALLOY SPHERICAL DOMES

J.J.V. Jeyasingh, K. Dhananjayan, G. Kothandaraman, P.P. Sinha
Mechanical Engineering Entity, Vikram Sarabhai Space Centre, Trivandrum-695022

B. Nageswara Rao
Structural Analysis and Testing Group, Vikram Sarabhai Space Centre, Trivandrum-695022

A. Chennakesava Reddy
Faculty of Mechanical Engineering, JNTU College of Engineering, Hyderabad - 500072

ABSTRACT

The gas pressure bulging of metal sheets has become an important forming method. As the bulging process progresses, significant thinning in the sheet material becomes obvious. A prior knowledge about non-uniform thinning in the product after forming helps the designer in the selection of initial blank thickness. This paper presents a simple analytical procedure for obtaining the thinning variation of superplastically formed Pb-Sn eutectic alloy spherical domes. It also addresses the issue of instability of deformation during gas pressure forming of superplastic sheets. With regard to fracture strain, the plastic behaviour of the spherical dome has been described in terms of the local effective stress and the effective strain. These quantities are equated to the uni-axial stress state. The limiting effective thickness strain is obtained utilizing the relations between the strain rate sensitivity index and the fracture strain. The results are found to be in good agreement with the measured failure strains.

KEYWORDS : Superplastic deformation; bulge forming; fracture strain

1. INTRODUCTION

Superplastic forming has become a promising processing technique in manufacturing industry. A number of processes are used for SPF, including Blow forming, Vacuum forming, Thermo forming, deep drawing and Die less drawing. Figure-1 shows the schematic diagram of the configuration for gas pressure forming. The top chamber holds the gas (air) at the predetermined pressure profiles through a pressure control system. The openings of the valves are adjusted through a proportional closed-loop control scheme. The bottom die contains a cavity of the same configuration as the required component. Using suitable spacers and fixtures, the top and bottom chambers were mounted coaxially onto the platen of a hydraulic press capable of applying constant loads for holding the blank during the entire forming operation. Spherical bulge forming is always used as a fundamental study of the superplastic formability. It involves many mechanical and geometrical features that are important to forming efficiency in bulging practices, such as damage evolution, ductility variation, strength level, thickness distribution, etc. Chokalingam et al.(1985) have made investigations related to the pressure forming(Sheet Thermo Forming) of the Pb-Sn eutectic alloy which is a model material that is superplastic at room temperature. While forming of Pb-Sn eutectic alloy superplastic sheet material into a spherical dome under constant effective strain rate through gas pressure forming, Khraisheh(2000) observed that failure occurs at a time near the peak value of the forming pressure

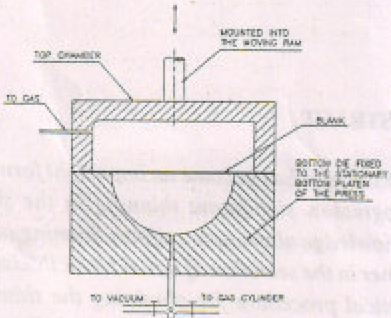

Fig.1 : Schematic diagram of the setup using gas pressure forming

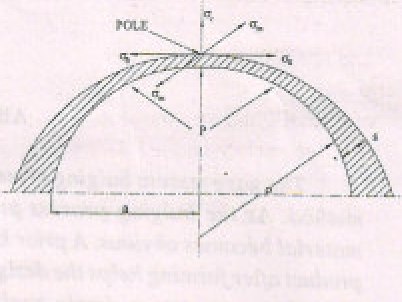

Fig.2 : Schematic representation of formed dome

profile constructed based on a mathematical model of Dutta et al.(1992). He reported that his simple instability analysis could not predict satisfactorily the limiting thickness strains at the pole of the spherical dome (where the maximum thinning takes place) due to the nature of anisotropic behaviour of Pb-Sn eutectic alloy. This paper addresses the issue of instability of deformation during gas pressure forming of superplastic sheets.

2. ANALYSIS

Thermomechanical constitutive equations of superplastic material describe the relationship between flow stress (σ), strain (ε), strain rate ($\dot{\varepsilon}$), temperature (T) and other microstructural quantities, usually the grain size (d). The mathematical relationship employs a number of model / material parameters (co-efficients) and material constants. Based on the phenomenological form of superplastic behaviour, the uniaxial flow stress σ is seen to be a strong function of inelastic strain-rate $\dot{\varepsilon}$ and a weak function of strain ε and grain size d. The material is assumed to be purely inelastic and incompressible. A functional form of the constitutive relationship is given by (Chandra, 2002)

$$\sigma = K(\dot{\varepsilon})^m \tag{1}$$

Where m strain-rate sensitivity.

Dutta et al.(1992) have assumed the following conditions in their analytical modeling of bulge forming. The material is isotropic and has no strain hardening (i.e., n = 0). The diaphragm is rigidly clamped at the periphery. The thickness (s) of the specimen is very small compared with the die radius (a), so that bending and shearing effects are negligible and membrane theory is assumed. The coefficient K and the strain rate sensitivity index (m) in equation (5) are constants. The bulge surface shape keeps to that of a part of a sphere (see Figure-2). There are three principal stresses at any point of the dome : The meridional stress (σ_m), the hoop stress (σ_θ), and the radial stress (σ_r, in the thickness direction). The value of σ_r is usually very small compared with σ_m or σ_θ and can be ignored. Then the stress state in the dome is $\sigma_m > 0$, $\sigma_\theta > 0$, $\sigma_r \approx 0$. A balanced biaxial stress state (i.e., $\sigma_m = \sigma_\theta$) exists at the dome apex.

Assuming plane stress, balanced biaxial stretching and volume constancy at the pole of the dome in free forming of a hemisphere, one can write the following relations :

$$\sigma_r = 0,\ \sigma_\theta = \sigma_m = \frac{P\rho}{2s},\ \sigma_e = \sigma_\theta \tag{2}$$

$$\varepsilon_\theta = \varepsilon_m, \qquad \varepsilon_\theta + \varepsilon_m + \varepsilon_r = 0,\ \varepsilon_e = -\varepsilon_r \tag{3}$$

Where P is the forming pressure ; • is the radius of curvature; σ_e is the von Mise's effective stress; and ε_e is the effective strain. Thickness strain (ε_r) is compressive in nature whereas the effective strain (ε_e) should be considered as positive.

The change in ε_r and ε_m can be expressed in terms of • and s as

$$d\varepsilon_m = \frac{d\rho}{\rho},\ d\varepsilon_r = \frac{ds}{s} \tag{4}$$

Integrating equation (4) and using equation (3), one can obtain:

$\ell n = \left(\frac{s}{s_0}\right) = \varepsilon_r = -\varepsilon_e$; and $\ell n = \left(\frac{\rho}{\rho_0}\right) = \varepsilon_m = \frac{1}{2}\varepsilon_e$. Here, s_0 is the initial blank thickness and ρ_0 is the reference radius of curvature of a spherical dome. For the case of a constant applied strain rate, $\dot{\varepsilon}_e$, the thickness of the bulged sheet (s) at a given forming time (t) under a constant effective strain rate ($\dot{\varepsilon}_e$) can be expressed as

$$s = s_0 \exp(-\dot{\varepsilon}_e t) \tag{5}$$

Following the analytical approach of Dutta et al.(1992), the initial and instantaneous areas (A_0 and A_i) of the deformed blank are written as : $A_0 = \pi a^2$ and $A_i = 2\pi\rho h = 2\pi\rho\left(\rho - \sqrt{\rho^2 - a^2}\right)$. Here, h is the height of the dome and a is the radius of the die. From constancy of volume (i.e. $A_0 s_0 = A_i s$), one can obtain the radius of curvature (•) in terms of s_0, a and s as

$$\rho = \frac{a}{2}\left\{\frac{s}{s_0}\left(1 - \frac{s}{s_0}\right)\right\}^{-\frac{1}{2}} \tag{6}$$

Where a is the radius of the die.

Using equations (2), (5) and (6), one can obtain the pressure-time relation as [Dutta et al.(1992)]

$$P = 4\frac{s_0}{a}\,\sigma_e \exp\left(-\frac{3}{2}\dot{\varepsilon}_e t\right)\sqrt{1-\exp(\dot{\varepsilon}_e t)} \tag{7}$$

The pressure-time curve under a constant effective strain rate at the pole of the dome can be obtained from equations (1) and (7). The peak pressure (P_{max}) at the time (t_p) can be obtained from equation (7) as

$$P_{max} = \left(\frac{3\sqrt{3}}{4}\right)\frac{s_0}{a}\,\sigma_e = \left(\frac{3\sqrt{3}}{4}\right)\frac{s_0}{a}\,K(\dot{\varepsilon}_e)^m, \quad t_p = \ell n\left(\frac{4}{3}\right)\frac{1}{\dot{\varepsilon}_e} \tag{8,9}$$

It is noted from equation (8) that the peak pressure is a function of the flow stress, initial sheet thickness and die radius. Since the flow stress value is dependent on the strain rate as per equation (1), P_{max} value also varies with the strain rate ($\dot{\varepsilon}_e$).

3. THICKNESS STRAIN ESTIMATES AT THE POLE OF THE DOME

Based on the empirical relation of Cheng (1996), the variation of the thickness in the hemi-spherical dome along the hoop direction is expressed in terms of 'ϕ' the angle between symmetry axis as

$$s = s_p + (s_e - s_p)\left(\frac{2\phi}{\pi}\right)^2 \tag{10}$$

where s_p and s_e are the thicknesses at the pole and equator of the spherical dome.

The relation between s_e and s_p through strain rate sensitivity index (m) and initial blank thickness (s_0) is obtained as

$$s_e = \left[(1-C)s_0^{\frac{1}{m}} + C s_p^{\frac{1}{m}}\right]^m \tag{11}$$

$$\text{where} \quad C = \left(\frac{\sqrt{3}}{2}\right)^{1+\frac{1}{m}}$$

Assuming Volume constancy, one can write

$$\pi a^2 s_0 = 2\pi a^2 \int_0^{\frac{\pi}{2}} s \sin\phi \, d\phi \tag{12}$$

Using equation (10) in equation(12), one can obtain

$$2\left[s_p + \frac{4}{\pi^2}(\pi-2)\,(s_e - s_p)\right] = s_0 \tag{13}$$

Using equation(11) in equation(13), one can obtain a non-linear equation for s_p. For the specified value of 'm', the thickness at the pole (s_p) can be determined by solving the resulting non-linear equation through Newton Raphson's iterative method.

The effective thickness strain can be obtained from,

$$\varepsilon_e = -\varepsilon_r = -\ell n\left(\frac{s_p}{s_0}\right) = \ell n\left(\frac{s_0}{s_p}\right) \tag{14}$$

Chokalingam et al.(1985) has conducted experiments on Pb-Sn eutectic alloy sheet of 1.3 mm thickness and 220 mm diameter at room temperature with the m value of 0.4. The diameter of the die opening was 80 mm and blank was subjected to an air pressure of 0.2 MPa. From equations(11) and (13), the thicknesses at the pole and equator of the spherical dome obtained are : $s_p = 0.402$ mm and $s_e = 0.926$ mm against the measured values of 0.36 and 0.92 respectively.

4. FRACTURE STRAIN ESTIMATIONS

Many papers have been published on studies of the plastic instability and the development of macroscopic necking of superplastic metals as well as on the relationship between the fracture strain and the parameters of the material. With regard to fracture strain, Ghosh and Ayres (1976) derived a relationship between the strain rate sensitivity index (m) and the fracture strain (e_f) by assuming that the strain in the imperfect region becomes infinite as the strain in the homogeneous region reaches the fracture strain (e_f) in their equation describing the force balance between the homogeneous and imperfect regions. Their result is,

$$e_f = \left(1 - f^{\frac{1}{m}}\right)^{-m} - 1 \tag{15}$$

where f is the inhomogeneity factor, which is the ratio of cross-sectional areas in the region of inhomogeneity to that outside it. With inhomogeneity factors of 0.998 and 0.999, the predicted limit strain from equation (15) compares reasonably with the experimental data on several materials collected by Woodford

(1969). It should be noted that the above results of plastic instability analyses are rigorously valid only when the state of stress is uniaxial. If the stress tensor deviates from a uniaxial one, the plastic behaviour of the component must be described in terms of the local effective stress (σ_e) and effective strain (ε_e). According to this concept, σ_e is numerically equal to the uniaxial stress that would produce the same effective strain rate as the one generated by the complex stress field. One of the attributes of superplastic flow that must be taken into consideration for the prediction of ductility is the case where grain coarsening is taking place (Jonas, 1982). In such situations, the current value of the strain rate sensitivity index (m) gradually diminishes with strain. Thus, the strain rate sensitivity index (m) can decrease from the vicinity of 0.5 to values as low as 0.25.

In his gas pressure forming experiments, Khraisheh(2000) observed that failure occurred at the pole of the dome, where the largest amount of thinning took place. Table-1 gives the details on the failure pressure recorded at different strain rates. The measured limiting thickness strains of Khraisheh(2000) during gas pressure forming of spherical domes presented in Table-1 at different applied constant strain rates are found to be almost constant. The average limiting thickness strain value is 1.6875.

Table 1 : Details on failure pressures and measured limiting thickness strains of Pb-Sn eutectic superplastically formed spherical domes at different strain rates

	Khraisheh(2000)				
Strain rate ε_e (s^{-1})	Actual Failure Pressure (MPa)	Forming time to failure (seconds)	Peak Pressure P_{max} (MPa)	Measured limiting thickness strain, ε_e *	Forming time to peak pressure(t_p) Eq. (9) (seconds)
1 x 10-4	0.512	2135	0.519	1.77	2877
3 x 10-4	0.687	935	0.691	1.66	959
6.5 x 10-4	0.768	550	0.775	1.66	443
1 x 10-3	1.013	195	1.039	1.66	288

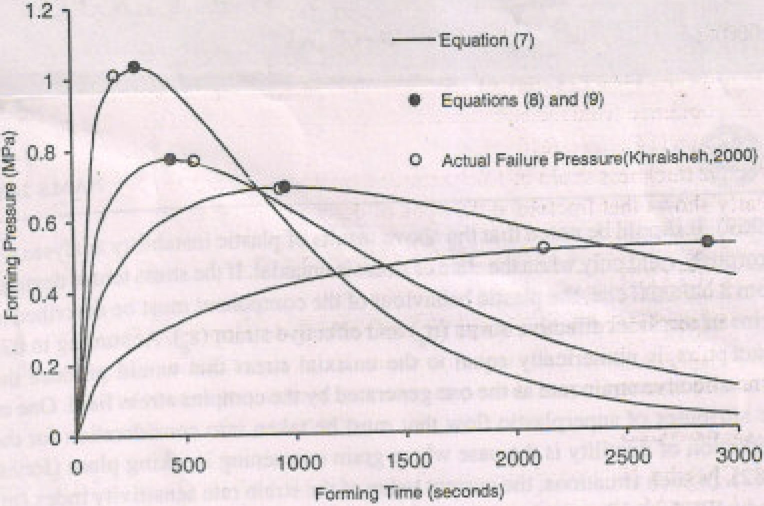

Fig. 3 : Forming pressure profiles of superplastic Pb-Sn eutectic alloy at different strain rates

Figure-3 shows the forming pressure profile generated using equation(7) for different strain rates. The forming pressure at failure marked in Figure-3 shows close to the peak pressure. The forming pressure at failure (P_f) was represented in terms of the applied strain rate ($\dot{\varepsilon}_e$) by Khraisheh(2000) : $P_f = 5.5(\dot{\varepsilon}_e)^{0.25}$ MPa. Since the peak pressure (P_{max}) is close to the forming pressure at failure (P_f) for the material considered by Khraisheh(2000), one can use this relation in equation (8) for P_{max} and equation (1) for σ_e, to get the strain rate sensitivity index (m). The value of m is found to be 0.25. With regard to fracture strain, the plastic behaviour of the spherical dome has to be described in terms of the local effective stress (σ_e) and the effective strain (ε_e). Since the failure occurred at the pole of the dome, where the largest amount of thinning took place, the effective stress and the strain at the pole of the dome are nothing but the hoop or meridional stress and the radial (thickness) strain respectively. These are to be equated to the uniaxial stress state. Using the value of inhomogeneity factor (f) of 0.995 as assumed by Lian et al (1986) , the values of fracture strain (ef) for the strain rate sensitivity index (m) of 0.25 from equations (15) is found to be 1.66414, which is found to be reasonably in good agreement with those of measured limiting thickness strains by Khraisheh (2000) .

For the present problem, m = 0.25 and s_0 = 1.27 mm, the thickness at the pole is obtained from the non-linear equation(13) as 0.2562 mm. Using this value in equation (14), the effective strain is found as 1.601, which is close to the failure effective thickness strain of 1.66 measured by Khraisheh(2000). This observation clearly shows that fracture at the apex of domes were observed by Khraisheh is mainly due to the low value of the strain rate sensitivity index (m = 0.25).

5. CONCLUSIONS

Superplastic fracture is a process of necking development. The influence of strain rate sensitivity index (m) on fracture strain (e_f) is examined considering the measured fracture strains during gas pressure forming of spherical domes.

REFERENCES

1. Chandra. N, "Constitutive behaviour of superplastic materials", Int. J. Non-Linear Mech., Vol.37, pp.461-484 (2002).
2. Cheng. J. H, "The determination of material parameters from superplastic inflation tests", J. Mater. Proc. Tech, Vol.58, pp.233-246 (1996)
3. Dutta, A and Mukherjee, A. K, "Superplastic Forming : an analytical approach", Mater. Sci. Eng., Vol.A157, pp.9-13 (1992).
4. Ghosh. A. K, and Ayres. R. A, "On reported anomalies in relating strain rate-sensitivity(m) to ductility", Metall. Trans. A., Vol.7, pp.1589-1591 (1976).
5. Jonas. J. J, "Superplastic forming of structural alloys", Paton, N. and Hamilton, C. H., eds., TMS-AIME, Warrendale, PA.(1982)
6. Khraisheh.M.K, "On the failure characteristics of superplastic sheet materials subjected to gas pressure forming", Scripta Materialia, Vol.42, pp.257-263 (2000).
7. Lian. J, and Baudelet. B, "Necking development and strain to fracture under uniaxial tension", Mater. Sci. Eng., Vol.84, pp.157-162 (1986).
8. Woodford. D. A, "Strain-rate sensitivity as a measure of ductility", Trans. of the ASM, Vol.62, pp.291-293(1969).