SPHERICAL DOME FORMATION BY TRANSFORMATION SUPERPLASTICITY OF TITANIUM MATRIX COMPOSITES

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ABSTRACT

Titanium alloys and Titanium matrix composites are useful materials in aerospace applications due to their high strength and stiffness, good corrosion resistance and low density. The gas pressure bulging of metal sheets has become an important forming method. As the bulging process progresses, significant thinning in the sheet material becomes obvious. This paper presents a simple analytical procedure for obtaining the dome height with respect to the forming time useful to the process designer for the selection of initial blank thickness as well as non-uniform thinning in the dome after forming. By thermally cycling through their transformation temperature range, coarse-grained, polymorphic materials can be deformed superplastically, owing to the emergence of transformation mismatch plasticity (or transformation superplasticity) as a deformation mechanism. This mechanism is investigated under biaxial stress conditions during thermal cycling of Titanium matrix composites(viz. Ti6Al4V/TiC and Ti6Al4V/TiB). For the transformation superplasticity, the strain rate sensitivity index is considered as unity. The transformation superplasticity constant in the constitutive relation is obtained from the measured dome height with respect to the forming time: The radius of curvature, thickness and height of the dome with respect to the forming time are obtained. The analytical results are found to be reasonably in good agreement with the test results.

KEYWORDS: Spherical Dome, Superplasticity, Composites

1. INTRODUCTION

Superplastic forming has become a promising processing technique in manufacturing industry. Several models for bulge forming have been established [1-6]. By thermally cycling through their transformation temperature range, coarse-grained, polymorphic materials can be deformed superplastically, owing to the emergence of transformation mismatch plasticity (or transformation superplasticity) as a deformation mechanism. Megam Frary et al. [7] have investigated under biaxial stress conditions during thermal cycling of unalloyed Titanium, Ti6Al4V, and their composites Ti/TiC, Ti6Al4V/TiC and Ti6Al4V/TiB. During gas-pressure dome bulging experiments, the dome height was measured as a function of time. It is noted that, the weakest material, CP-Ti deformed most rapidly, while the Ti6Al4V based materials deformed more slowly due to the longer thermal cycle times and higher creep resistance during a thermal cycle, transformation superplasticity contributes to the deformation only when the phase transformation is occuring. At all other times, the material deforms only under the action of external stress by a typical creep mechanism (eg. Dislocation creep). Since these two mechanisms operate at different times the cycle, they contribute to the total deformation independently, and it is reasonable to add their contributions :

$$\dot{\varepsilon} = \dot{\varepsilon}_{creep} + \dot{\varepsilon}_{TSP} = K_{creep}\,\sigma^{n} + K_{TSP}\,\sigma \qquad (1)$$

Where $\dot{\varepsilon}$ is the strain rate, σ is the flow stress, n is the stress exponent, K_{TSP} is the transformation superplasticity constant and K_{creep} is the dislocation creep constant. It is noted from the experimental studies on bi-axial deformation of composite materials that transformation superplasticity is the only operative deformation mechanism for the Ti6Al4V composites and there is no contribution from creep. Hence the constitutive relation for the present problem becomes

$$\dot{\varepsilon} = K_{TSP}\,\sigma \qquad (2)$$

For transformation superplasticity, the strain rate sensitivity index from equation(2) is found to be unity. This paper presents a simple analytical procedure for obtaining the dome height with respect to the forming time useful to the process designer for the selection of initial blank thickness as well as non-uniform thinning in the dome after forming.

2. ANALYSIS

Figure-1 shows the apparatus of biaxial doming experiments carried out by Megam Frary et al.[7]. Dutta and Mukherjee[2] have assumed the following conditions in their analytical modeling of bulge forming. The material is isotropic. The diaphragm is rigidly clamped at the periphery.

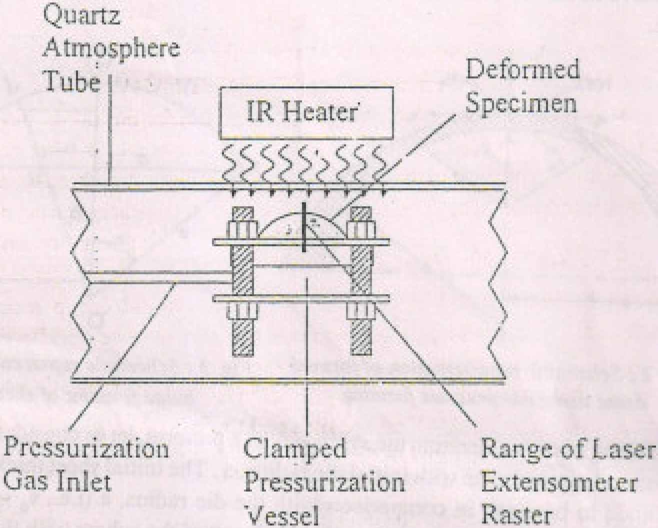

Fig.1 : Schematic of the experimental biaxial gas-pressure apparatus

The thickness (s) of the specimen is very small compared with the die radius (a), so that bending and shearing effects are negligible and membrane theory is assumed. The coefficient K and the strain rate sensitivity index (m) in the constitutive equation: $\sigma = K(\dot{\varepsilon})^m$, are constants. The bulge surface shape keeps to that of a part of a sphere (see Figure-2). There are three principal stresses at any point of the dome : The meridional stress (σ_m), the hoop stress (σ_θ), and the radial stress (σ_r, in the thickness direction). The value of σ_r is usually very small compared with σ_m or σ_θ and can be ignored. Then the stress state in the dome is $\sigma_m > 0$, $\sigma_\theta > 0$, $\sigma_r \approx 0$. A balanced biaxial stress state (i.e., $\sigma_m = \sigma_\theta$) exists at the dome apex.

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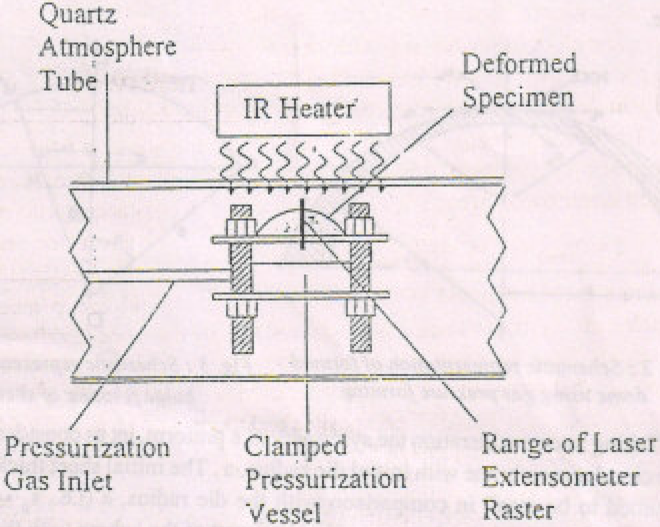

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The hoop, meridional and thickness strains at M∗ are,

$$\varepsilon_\theta = \ell n\left(\frac{\rho\alpha}{a}\right) \tag{5}$$

$$\varepsilon_m = \ell n\left(\frac{2\pi r}{2\pi r_0}\right) = \ell n\left(\frac{\rho\alpha}{a}\ \frac{\sin\phi}{\phi}\right) \tag{6}$$

$$\varepsilon_r = \ell n\left(\frac{s_\phi}{s_0}\right) \tag{7}$$

where s_ϕ is the thickness at M∗. α is the angle subtended by the apex and the edge of the dome (see Figure-3) and φ is the angle between the symmetry axis and the dome radius, drawing to the point under consideration. Jeyasingh et al.[5,6] have developed the following equations for the variation in the thickness as a function of position in the dome :

$$\frac{s_\phi}{s_0} = \left(\frac{s_p}{s_0}\right)^\delta \tag{8}$$

where $\delta = \frac{\beta^{\frac{1}{m}}}{\gamma}$, $\beta = \frac{\sqrt{3(1+\kappa+\kappa^2)}}{(\kappa+2)}$, $\gamma = \frac{2}{\sqrt{3}}\frac{\sqrt{1+\kappa+\kappa^2}}{(\kappa+1)}$

and $\kappa = 1 + \frac{\left(\frac{\sin\phi}{\phi}\right)}{\ell n\left(\frac{\alpha}{\sin\alpha}\right)}$

Here s_0 is the initial blank thickness, s_p and s_e are thicknesses at the pole and edge of the dome. References [4,6] suggests an empirical relation for the variation of the thickness (s_ϕ) as

$$s_\phi = s_p + (s_e - s_p)\left(\frac{\phi}{\alpha}\right)^2 \tag{9}$$

Assuming the volume constancy, we derived

$$s_0 = 2\,\mathrm{Cosec}^2\alpha\,[sp\,I_0 + (s_e - s_p)\,I_1] \tag{10}$$

$$\text{where } I_0 = 1 - \cos\alpha \text{ and } I_1 = -\cos\alpha + \frac{2\sin\alpha}{\alpha} - \left(\frac{2\sin\frac{\alpha}{2}}{\alpha}\right)^2$$

Substituting $\phi = \alpha$ in equation (8), one can get s_e in terms of s_p and s_0.

Using s_e in equation (10), one can obtain s_p by solving the resulting non-linear equation through Newton Raphsons iterative method. After determining s_p, the thickness profile of the dome can be estimated from equation (9).

Using equations (5),(6),(7) in equation (4), one can obtain the variation in thickness as a function of position in the dome from [3]

$$\frac{s_\phi}{s_0} = \left(\frac{\sin\alpha}{\alpha}\right)^2 \frac{\phi}{\sin\phi} \qquad (11)$$

It should be noted that s_p from equation (10) is a function of the strain rate sensitivity index m where as equation (11) is independent of m. To examine the adequacy of equations (8) and (11), a comparative study is made in Table-1 considering the measured data of Megam Frary et al.[7]. and Dutta and Sharma [8].

Table 1 : Comparison of analytical and experimental Pole thickness of spherical domes after superplastic forming

Material	Experimental Process parameters					Pole thickness, S_p (mm)	
	m	a (mm)	s_0 (mm)	α (Deg.)	s_p (mm)	Equation (10)	Equation (11)
Ti6Al4V/TiC [7]	1.0	24	1.51	58.91	1.179	1.094	1.048
Ti6Al4V/TiB [7]	1.0	24	1.59	41.87	1.372	1.359	1.326
Ti6.3Al2.7Mo1.7Zr [8]	0.85	15.15	2.7	90	1.194	1.146	1.094

It is noted that equation (11) is valid for the strain rate sensitivity index (m) close to unity. Though equation (10) gives the pole thickness (s_p) through an iterative process close to the measured thickness, equation (11) gives s_p directly and accurate for (m) close to unity being followed in the present analysis.

Combining equation (2), equation (3) and equation (11), one can write the following equation to obtain α for the specified constant pressure at any instant of time:

$$\frac{d\alpha}{dt} = K_{TSP}\left(\frac{p\rho}{4s_p}\right)\left(\frac{1}{\alpha} - \mathrm{Cot}\,\alpha\right)^{-1} \tag{12}$$

$$\text{where } s_p = s_0\left(\frac{\mathrm{Sin}\,\alpha}{\alpha}\right)^2 \text{ and } \rho = \frac{a}{\mathrm{Sin}\,\alpha}$$

Equation (12) is solved specifying the initial condition :

$$\alpha = \text{at } t = 0 \tag{13}$$

Equations (12) and (13) represent a non-linear differential equation for the rate of change in α, as a function of the forming pressure, initial sheet thickness (s_0), die radius (a) and the transformation superplasticity constant K_{TSP}. For a very small forming time, one can find the following approximate solution to equations (12) and (13):

$$\alpha = \left\{\frac{9}{4} K_{TSP}\left(\frac{pa}{s_0}\right)t\right\}^{\frac{1}{3}} \tag{14}$$

The height of the dome at each instant of time can be obtained from,

$$h = \frac{a}{\mathrm{Sin}\,\alpha}(1 - \mathrm{Cos}\,\alpha) \tag{15}$$

the thinning of the dome can be directly obtained from equation(11) by specifying ϕ values between 0 and α.

Equation (12) is solved by finite difference method with the step size, Δt = 100 sec.

From the biaxial dome experiments of Megam Frary et al.[7], the transformation superplasticity constant K_{TSP} is $2.55 \times 10^{-12}\ Pa^{-1}\ s^{-1}$ for titanium matrix composites : Ti6Al4V/TiC and Ti6Al4V/TiB. Using this value in equation (12) and solving, we obtain the dome height with respect to the forming time. Figure-4 shows the comparison of analytical and experimental results of the dome height with respect to the forming time. The analytical results are found to be reasonably in good agreement with test results.

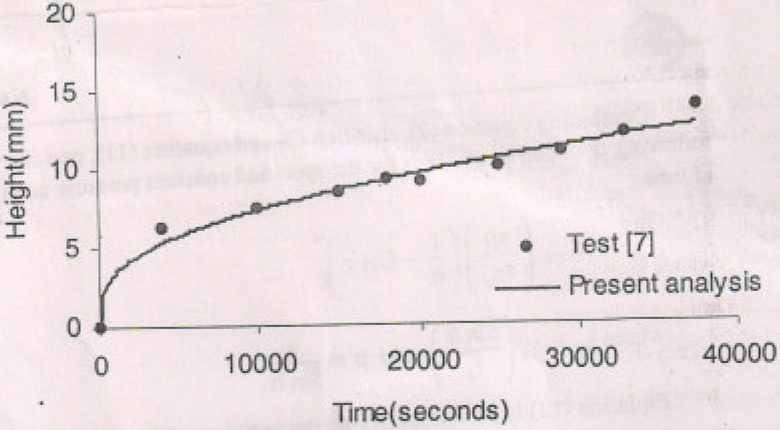

Fig. 4(a) : Dome height as a function of time for Ti6Al4V/TiC

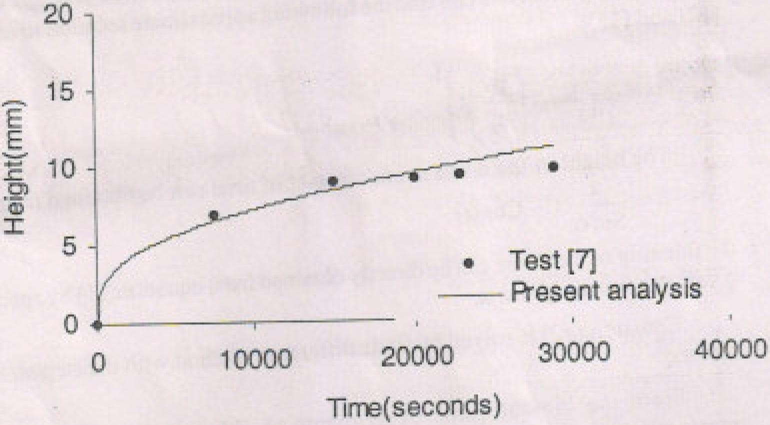

Fig.4 (b) : Dome height as a function of time for Ti6Al4V/TiB

3. CONCLUSION

A simple analytical procedure is followed for obtaining the dome height with respect to the forming time. This procedure is validated from the biaxial dome experimental results of titanium matrix composites [7]. Once we know the pole thickness of the dome, the effective thickness strain can be estimated directly from equations (4) and (7).

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