Materials Sciences Research Journal 2008, Volume 2, Issue 3/4 ISSN 1935-2441
pp.191-206

DEVELOPMENT OF A DUCTILE FRACTURE CRITERION IN COLD FORMING

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ABSTRACT

The Knowledge of metal flow and strain at fracture is most importance to the plastic damage in metal forming. Failure in metal working usually occurs as ductile fracture, rarely as brittle fracture. In terms of metal forming, the propagation of cracks is of little interest, since the main issue is to avoid their initiation. Therefore, the main effort has historically been placed not in developing a full mechanics analysis of ductile cracking, but simply on establishing criteria for predicting the fracture initiation sites and the level of deformation at which the crack will occur. A comparative study has been made on widely used criteria for predicting the occurrence of fracture in metal forming processes by using the upset test data of cylindrical specimens. A simple criterion, which is based on integrals of stress function, is proposed and its applicability is validated through the existing test data on commercial purity aluminium. The finite element analysis results of the complex geometries and loadings can be used in the proposed criterion for predicting the occurrence of defects. The proposed criterion acts as a limiting condition for the defects / cracks initiation under cold working conditions and takes into account the tensile stress ratio and tri-axiality.

Keywords: Cold Workability, Metal forming, Ductile fracture criteria, Effective failure strain, Mean stress, Aluminium alloy.

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1. INTRODUCTION

Forming and forging processes are among the oldest and most important of materials related technologies. New technologies focus on the development and widespread use of thermo-mechanical processing of materials. Metal forming process uses a remarkable property of metals viz. the ability to flow plastically in the solid state without concurrent deterioration of the properties. These are classified into hot and cold working processes. In most cases of manufacturing, cold working is done at room temperature. In some cases the working is done at intermediate temperature (warm working) that will provide increased ductility and reduced strength, but will be below recrystallisation temperature. In hot working of metals (that is temperatures above the recrystallisation temperature) the influence of strain on flow stress is insignificant, and the influence of strain rate (rate of deformation) becomes increasingly important. Conversely, at lower temperatures, the effect of strain rate on flow stress is known to be negligibly small and the effect of strain on flow stress (strain hardening) is most important.

The term workability is usually defined as the relative use with which a metal or alloy can be shaped through plastic deformation. The evaluation of workability involves both measurement of the resistance to deformation (strength) and the amount of plastic deformation before fracture (ductility). Therefore a complete description of the workability of a material is specified by its flow stress dependence on processing variables (strain rate, die-temperature, pre-heat temperature, etc.), its failure behaviour and the metallurgical factors that control the microstructure of the material. Edge cracks in rolling, internal cracks in extrusion, tears in sheet forming, and laps and surface cracks in forging are just a few of the wide variety of undesirable defects that may occur during metal forming operations. The majority of these defects are initiated due to localization of plastic shear and subsequent ductile failure.

It is evident in the context of the bulk metal working that ductile fracture is of significance since it represents the limit of plastic flow. If this limit is exceeded, the integrity of the work piece is destroyed by the initiation of cracks in the metal. Changes in the metal working operation to avoid fracture are much cheaper in the planning stage than retrospective changes necessitated after fracture has been observed in practice.

This paper deals with the ductile fracture criteria in cold working by examining some of the criteria commonly used to predict the initiation of fracture through test data on commercial purity aluminium. A limiting condition is proposed which is based on integrals of stress functions. The applicability of this condition is demonstrated by correlating the predicted fracture strains with experimental data.

2. LIMITING CONDITION FOR FRACTURE INITIATION

A ductile fracture criterion is a theoretical law designed to predict how far a metal can be deformed without cracks being formed in the work piece. Ductile fracture in metals is, generally to be governed by the formation of voids on the micro scale. This evolution of a void is characterized by three stages viz., nucleation, growth and coalescence. The nucleation of voids occurs near second phase particles, inclusions, dislocation pileups or other imperfections present in the material. Deformation of the material causes a concentration of stress and strains in the vicinity of such imperfections, which on reaching a critical value during deformation result in the nucleation of void. The voids can grow under the influence on continuing the plastic deformation. The rate of growth of voids is governed by the deformation history and the stress state applied. At a certain stage in the process of void growth the deformation will be localized between neighboring voids, causing these ligaments to fail and the voids to coalesce. The failure mechanism has often been attributed to the development of the smaller micro voids inside the ligaments. The onset of coalescence defines the initiation of a ductile crack. The failed ligament can be observed on fracture surface in the form of the so-called dimples or shear lips.

In order to predict ductile fracture in metals, an extensive research effort has been devoted to the modeling of various stages of void evolution [1-3]. This effort has resulted in sophisticated extensions of the original model for porous plasticity proposed by Gurson [4]. A general class of fracture criteria is used for a ductile fracture is stated to initiate when an integral expressions which is a function of the deformation and loading history, reaches a critical value given by the material parameter C [5,6]:

$$\int_{0}^{\bar{\varepsilon}_f} f(\sigma)\, d\varepsilon \geq C \tag{1}$$

in which $\bar{\varepsilon}_f$ denotes the effective plastic failure strain

The criterion in equation (1) postulates the condition for crack growth to be governed by a threshold value C, which should be considered as a material parameter. The Kernel function $f(\sigma)$ reflects the influence of stress-strain on the degradation of material and is usually strongly related to the tri-axiality. A large number of proposals for $f(\sigma)$ have been published [7]. The experimental determination of this parameter C is the key issue in literature. No example has been found where the initiation of ductile fracture is accurately predicted for different loading situation with one critical parameter C. Generally, the experimental determination of C is performed under loading conditions comparable to the loading conditions in the desired applications. The quantification of the right hand side of the equation (1) is referred to as the characterization of the fracture model.

An experiment is needed to be chosen that allows the identification of ductile fracture initiation. Accordingly, a numerical simulation of that experiment is executed up to the moment of ductile fracture initiation.

During the numerical simulation, the left hand side of the equation (1) is compared as a field variable. As the simulation reaches the experimentally determined point of ductile fracture initiation, the parameter C is quantified to be the occurring limit of the integral. A check should confirm that the location of the maximum agrees with the experimental position of ductile fracture initiation. When ductile fracture initiation model is characterized, it can be applied to a forming operation with an arbitrary geometry.

To control initiation and the growth of voids, the fracture potential is defined as:

$$p = \frac{\int_0^{\bar{\varepsilon}_f} f(\sigma) d\varepsilon}{C} \tag{2}$$

which represents the damage locally accurate in the material and

$$p \to 1 \tag{3}$$

becomes the limiting condition at the initiation of fracture.

3. The Kernel Function $f(\sigma)$

For a criterion to be successful in predicting workability in a bulk forming process, it should be capable of determining the amount of deformation before fracture as well as the fracture initiation site. Many criteria have been developed to predict ductile fracture. Some of them are based purely on experimental data (i.e., empirical) while others are developed from theoretical foundations. An empirical workability criterion was suggested by Kuhn et al. [8-10], which states that the axial and circumferential strains at fracture provide a measure of workability, since they fall on a straight line for given temperature, strain rate and microstructure. Realizing that plastic instability and ductile fracture are much influenced by stress field, several investigators attempted to develop stress based fracture criteria. Vujovic and Shabaik [11] claimed that fracture or failure would occur when effective strain at any point in deformation region reaches a critical value. They expressed the effective strain as a function of the ratio of hydrostatic or mean normal stress and the effective stress. Void growth, which is an accepted cause of ductile fracture has been used at the microscopic level, by McClintock and others [12-14], to formulate fracture criteria. Localized thinning due to inhomogenity, which is quite evident in sheet metal forming, has been used (Lee and Kuhn [9]), to develop a fracture criterion for bulk forming operations. Other criteria based on integrals of stress and strain functions have been developed from macroscopic considerations by Cockroft and Latham and others [15-17]. Techniques of continuum damage mechanics have also been used to predict ductile fracture [18].

In the absence of a reliable quantitative model for estimating the ductility of a material undergoing large plastic flow in metal forming or metal working process, a number of phenomenological models for metal processing ductility have been developed as above. However, no general theoretical means of predicting the occurrence of this type of failure has been advanced and the successful avoidance of fracture has been largely a matter of empirical practice. Some of the Kernel functions (i.e., $f(\sigma)$ in equation (1)) commonly used to predict ductile fracture are presented below.

$$f(\sigma) = \bar{\sigma} \quad \text{(Freudenthal [19])} \tag{4}$$

$$f(\sigma) = \sigma_{\max} \quad \text{(Cockroft and Latham [15])} \tag{5}$$

$$f(\sigma) = \frac{2\sigma_{\max}}{3(\sigma_{\max} - \sigma_H)} \qquad \text{(Brozzo et al. [16])} \qquad (6)$$

$$f(\sigma) = \frac{\sigma_{\max}}{\bar{\sigma}} \qquad \text{(Oh et al. [20])} \qquad (7)$$

$$f(\sigma) = 1 + A\frac{\sigma_H}{\bar{\sigma}} \qquad \text{(Oyane et al. [21])} \qquad (8)$$

$$f(\sigma) = \frac{\sigma_{\max}}{\bar{\sigma}} + B\frac{\sigma_H}{\bar{\sigma}} \qquad \text{(Murty et al. [22,23])} \qquad (9)$$

where $\bar{\sigma}$ is the effective stress ; $\sigma_{\max}$ is the maximum tensile stress ; σ_H is the hydrostatic or mean normal stress ; *A* and *B* are material constants.

4. UPSETTING TEST OF CYLINDRICAL SPECIMENS

A useful experimental method for studying the ductile fracture problem is based on the uni-axial compression of a cylindrical specimen between flat compressive platens. Compression specimens of initial height 'H_0' and diameter 'D_0' can be compressed between platens that carry various types of interface lubrication. This enables to get various friction conditions between the specimens and the platens which vary from approximately zero friction, where the specimen remains cylindrical and exhibits only homogenous deformation to un-lubricated conditions, where "sticking friction" effects cause a severe non-uniform 'barreling' mode of deformation (see Figure-1). The equatorial free surface is the region where ductile fracture initiates. The barreled compression specimen therefore has the advantage over the uni-axial tension specimen of allowing direct non destructive observations of the progressive ductile fracture process. Thus, the upsetting of a small cylinder at room temperature is one of the most widely used workability test.

The strain state usually consists of a circumferential tensile stress and an axial compressive stress, although an axial tensile stress may develop when barreling is severe. The surface strains measured at the onset of fracture for a wide range of test conditions enable, construction of a fracture or "forming limit diagram" for the material. In the absence of friction, the tensile strain is equal to one half of the compressive strain. Increasing the frictional constraint causes bulge severity to increase, which in turn increases the tensile strain and decreases the compressive strain. Beginning with the strains ratio of one half for frictionless deformation, the strain path slope increases with increasing friction. A wide range of strain paths can be produced at the free surfaces of cylindrical specimen permitting evaluation of the fracture or forming limit of the material [24-28]. Generally, strain path curves are generated from the measurements of the axial and circumferential plastic principal strains at the equatorial free surface of upsetting cylindrical specimens of various aspect ratios, H_0/D_0. The end points for all the strain paths, which represent cracking or fracture, for a material give the forming limit diagram for the material under consideration. Computation of induced stresses from the measured strains is essential for evaluation of material parameter *C* in equation (1).

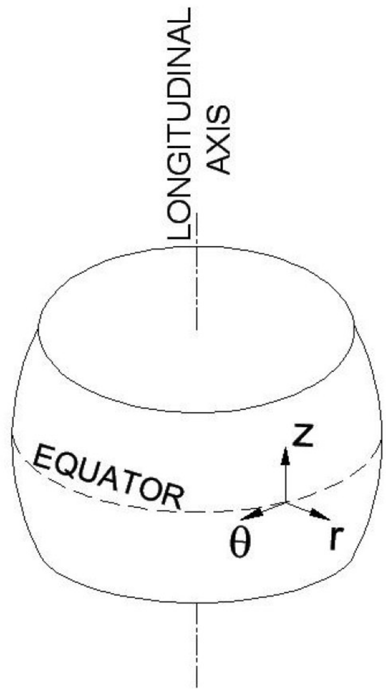

Figure1. Barreled compression specimen.

5. Computation of Stresses from Measured Strains

The components of the stress and strain at free surface of a compressed cylinder can be written as [29,30]

$$d\varepsilon_z = \frac{d\bar{\varepsilon}}{\bar{\sigma}}\left(\sigma_z - \frac{1}{2}\sigma_\theta\right) \tag{10}$$

$$d\varepsilon_r = -\frac{d\bar{\varepsilon}}{2\bar{\sigma}}\left(\sigma_\theta + \sigma_z\right) \tag{11}$$

$$d\varepsilon_\theta = -\frac{d\bar{\varepsilon}}{\bar{\sigma}}\left(\sigma_\theta - \frac{1}{2}\sigma_z\right) \tag{12}$$

where the effective strain increment is given by

$$d\bar{\varepsilon} = \sqrt{\frac{2}{3}\left(d\varepsilon_z^{\,2} + d\varepsilon_r^{\,2} + d\varepsilon_\theta^{\,2}\right)} \tag{13}$$

and $\bar{\sigma}$ is the effective stress. The subscripts z, r and θ represent the quantities along the axial, radial and circumferential directions of the cylinder respectively. For constant volume conditions, the summation of strain increments is zero i.e.,

$$d\varepsilon_z + d\varepsilon_r + d\varepsilon_\theta = 0 \tag{14}$$

Equations (10) to (12), satisfy the above constant volume condition (14). Defining the strain increment ratio

$$\beta = \frac{d\varepsilon_\theta}{d\varepsilon_z} \tag{15}$$

and using equation (14) in (13), one can get,

$$d\bar{\varepsilon} = -\frac{2}{\sqrt{3}}\sqrt{\beta^2 + \beta + 1}\, d\varepsilon_z \tag{16}$$

By integrating equation (16) becomes

$$\bar{\varepsilon} = -\frac{2}{\sqrt{3}}\int_0^{\varepsilon_z}\sqrt{\beta^2 + \beta + 1}\, d\varepsilon_z \tag{17}$$

Substituting the axial fracture strain ε_{zf} to the upper limit of integration in equation (17) one can get the effective fracture strain $\bar{\varepsilon}_f$. From the measured strain components ε_z and ε_θ on the outer surface of the compressed cylinder the effective strain $\bar{\varepsilon}$ can be computed from equation (17). The radial stress component σ_r is zero at the outer surface of the cylinder. For many engineering materials, the relationship between the effective stress $\bar{\sigma}$ and the effective strain $\bar{\varepsilon}$ is given in the form of a power law equation:

$$\bar{\sigma} = K(\bar{\varepsilon})^n \tag{18}$$

where *K* is the strength co-efficient and *n* is the strain hardening exponent.

The mean or the hydrostatic stress σ_H is given by

$$\sigma_H = \frac{\sigma_r + \sigma_\theta + \sigma_z}{3} = \frac{\sigma_\theta + \sigma_z}{3} \tag{19}$$

since $\sigma_r = 0$ at the outer surface of the cylinder.

By adding equations (10) and (12) and using equations (15),(16) and (19) in the resulting equation, one can get

$$\frac{\sigma_H}{\bar{\sigma}} = -\frac{1}{\sqrt{3}} \frac{(\beta+1)}{\sqrt{\beta^2+\beta+1}} \tag{20}$$

Eliminating σ_z from equations (10) and (12) and using equations (16) and (19) in the resulting equation, one can get

$$\frac{\sigma_\theta}{\bar{\sigma}} = -\frac{1}{\sqrt{3}} \frac{(2\beta+1)}{\sqrt{\beta^2+\beta+1}} \tag{21}$$

From equations (19) to (21), one can write

$$\frac{\sigma_z}{\bar{\sigma}} = -\frac{1}{\sqrt{3}} \frac{(\beta+2)}{\sqrt{\beta^2+\beta+1}} \tag{22}$$

Equations (20) to (22) give the stress components $\sigma_{\theta r}$ and σ_z and the mean stress σ_H in terms of effective stress $\bar{\sigma}$ and the strain increment ratio β . Using the determined value of $\bar{\varepsilon}$ in equation (18), the effective stress $\bar{\sigma}$ can be obtained. Once the effective stress $\bar{\sigma}$ is known for the specified values of ε_z and ε_θ, the mean stress components σ_m , $\sigma_{\theta r}$ and σ_z can be determined directly from equations (20) and (22). In all the above computations, accurate evaluation of the strain increment ratio β and the effective strain $\bar{\varepsilon}$ from equations (15) and (17) using the measured strain data of ε_z and ε_θ upto the fracture is essential. The axial and circumferential plastic principal strains ε_z and ε_θ measured at the equatorial free surface of upsetting cylindrical specimens of various aspect ratios for lubricated and un lubricated platens are generally fitted in a cubic polynomial of the form:

$$\varepsilon_\theta = a_1 \varepsilon_z{}^3 + a_2 \varepsilon_z{}^2 + a_3 \varepsilon_z \tag{23}$$

Differentiating equation (23) with respect to ε_z one can get the strain increment ratio as

$$\beta = \frac{d\varepsilon_\theta}{d\varepsilon_z} = 3a_1 \varepsilon_z{}^2 + 2a_2 \varepsilon_z + a_3 \tag{24}$$

The stress ratio $\frac{\sigma_H}{\bar{\sigma}}$, $\frac{\sigma_z}{\bar{\sigma}}$ and $\frac{\sigma_\theta}{\bar{\sigma}}$ in equations (20) to (22) are evaluated for different values of $\varepsilon_z \in \left|0, \varepsilon_{zf}\right|$ by computing β from equation (24). The definite integral in equation (17) is evaluated through a five point Gauss quadratures.

6. EVALUATION OF THE MATERIAL PARAMETER *C*

Using equations (4) to (9) for the Kernel functions in equation (1), the material parameter '*C*' is evaluated from the measured strain data of ε_z and ε_θ upto the fracture of the cylindrical specimens under axial compression. The maximum tensile stress in equations (5) to (9) is nothing but the circumferential stress (σ_θ) for the case of cylindrical specimens under axial compression. Using equations (16), (18), (20) and (21) in equation (1), one can obtain the parameter '*C*' for each fracture criterion is as follows.

$$C = \int_0^{\bar{\varepsilon}_f} \bar{\sigma}\, d\bar{\varepsilon} = \frac{k\left(\bar{\varepsilon}_f\right)^{n+1}}{n+1} \qquad \text{(Freudenthal [19])} \qquad (25)$$

$$C = \int_0^{\bar{\varepsilon}_f} \sigma_\theta\, d\bar{\varepsilon} = \frac{2}{3}\int_0^{\varepsilon_{zf}} (2\beta+1)\bar{\sigma}\, d\varepsilon_z \qquad \text{(Cockroft and Latham [15])} \qquad (26)$$

$$C = \int_0^{\bar{\varepsilon}_f} \frac{2\sigma_\theta}{3(\sigma_\theta - \sigma_m)} d\bar{\varepsilon} = -\frac{4}{3\sqrt{3}}\int_0^{\varepsilon_{zf}} (2+\frac{1}{\beta})\sqrt{\beta^2+\beta+1}\, d\varepsilon_z$$

(Brozzo et al. [16]) (27)

$$C = \int_0^{\bar{\varepsilon}_f} \frac{\sigma_\theta}{\bar{\sigma}} d\bar{\varepsilon} = \frac{2}{3}\int_0^{\varepsilon_{zf}} (2\beta+1) d\varepsilon_z = \frac{2}{3}\left(2\varepsilon_{\theta f} + \varepsilon_{zf}\right) \qquad \text{(Oh et al. [20])} \qquad (28)$$

$$C = \bar{\varepsilon}_f + A\int_0^{\bar{\varepsilon}_f} \frac{\sigma_H}{\bar{\sigma}} d\bar{\varepsilon} = \bar{\varepsilon}_f + \frac{2}{3}A\int_0^{\varepsilon_{zf}} (\beta+1) d\varepsilon_z = \bar{\varepsilon}_f + \frac{2}{3}A\left(\varepsilon_{\theta f} + \varepsilon_{zf}\right)$$

(Oyane et al. [21]) (29)

$$C = \int_0^{\bar{\varepsilon}_f} \left(\frac{\sigma_\theta}{\bar{\sigma}} + B\frac{\sigma_H}{\bar{\sigma}}\right) d\bar{\varepsilon} = \frac{2}{3}\int_0^{\varepsilon_{zf}} \{(2\beta+1) + B(\beta+1)\} d\varepsilon_z = \frac{2}{3}\{(2\varepsilon_{\theta f} + \varepsilon_{zf}) + B(\varepsilon_{\theta f} + \varepsilon_{zf})\}$$

(Murty et al. [22,23]) (30)

All the above integrals are evaluated using a five point Gauss quadratures.

7. RESULTS AND DISCUSSION

Experimental results [27] of cold upsetting of aluminium cylindrical specimens are considered in the present study to evaluate the material parameter '*C*' in equation (1). The measured axial and circumferential principal strains ε_z and ε_θ at the equatorial free surface of the upsetting cylindrical specimens for various aspect ratios for lubricated and un-lubricated platens are fitted in a cubic polynomial as in equation (23). The constants a_1, a_2, a_3 in equation (23), to represent ε_θ in terms of ε_z, for four strain paths are presented in Table-1. The material constants K and n in the power-law relation (18) between the effective stress and strain are : K=180 MPa and n=0.20.

The strain increment ratio (β) for the specified axial strain (ε_z) can be obtained from equation (15). The effective strain ($\bar{\varepsilon}$) is obtained from equation (17) after integration through a five point Gauss quadratures. The corresponding effective stress ($\bar{\sigma}$) is obtained from the power-law relation (18). The mean or the hydrostatic stress (σ_H), the circumferential stress (σ_θ) and the axial stress (σ_z) are obtained from equations (20) – (22) for the specified (ε_z). By specifying the measured axial failure strain (ε_{zf}) in equation (17), the effective fracture strain ($\bar{\varepsilon}_f$) is obtained. It is noted from the results in Table-2 that $\bar{\varepsilon}_f$ is varying with ε_{zf}.

Table 1. Constants in equation (23) to represent ε_θ in terms of ε_z for commercial purity Aluminium [27]

Strain Path	a_1	a_2	a_3
I	-85.015	-24.1	-2.93
II	-31.7	-10.8	-2.0
III	-5.3	-1.0	-0.91
IV	-1.3	-0.05	-0.6

Table 2. The axial, circumferential and effective failure strains (ε_{zf}, $\varepsilon_{\theta f}$ and $\bar{\varepsilon}_f$) of the upsetting aluminium cylindrical specimens

Strain Path	Failure Strains		
	Axial (ε_{zf}) Ref.[27]	Circumferential ($\varepsilon_{\theta f}$) Equation (23)	Effective ($\bar{\varepsilon}_f$) Equation (17)
I	-0.253	0.5754	0.6175
II	-0.321	0.5777	0.6161
III	-0.420	0.5985	0.6431
IV	-0.589	0.6017	0.7210

The material parameter '*C*' obtained from equations (23) to (26) by specifying the failure strains of Table-2 are presented in Table-3. The material parameter "*C*" is found to vary with the failure strain. The average value of '*C*' is considered in equation (2) while evaluating the fracture potential (p). Using the failure strain data of Table-2 in equation (27), the constant A in the kernel function of Oyane et al. [21] criterion and the material parameter '*C*' are determined through least square curve fit.

Table 3. Material parameter '*C*' obtained using the Kernel functions (4) to (7) in equation (1)

Strain Path	Material parameter '*C*'			
	Freudenthal criterion Eq.(25)	Cockroft and Latham criterion Eq. (26)	Brozzo et al. Criterion Eq. (27)	Oh et al. criterion Eq. (28)
I	84.11	83.34	0.6353	0.5986
II	83.89	77.86	0.5853	0.5562
III	88.32	74.47	0.5462	0.5180
IV	101.3	62.23	0.4582	0.4096
Average	89.41	74.48	0.5563	0.5206

Table 4. Comparison of experimental and predicted axial failure strains ε_{zf} for commercial purity Aluminium

Strain Path	Experimental ε_{zf} [27]	Theoretical Results			Relative Error (%)
		ε_{zf}	$\varepsilon_{\theta f}$	$\bar{\varepsilon}_f$	
Freudenthal Criterion					
I	-0.253	-0.258	0.612	0.657	-2.0
II	-0.321	-0.327	0.610	0.650	-1.9
III	-0.420	-0.424	0.611	0.656	-1.0
IV	-0.589	-0.555	0.538	0.657	5.9
Cockroft and Latham Criterion					
I	-0.253	-0.248	0.541	0.580	2.0
II	-0.321	-0.318	0.562	0.600	0.9
III	-0.420	-0.420	0.598	0.643	0.0
IV	-0.589	-0.630	0.684	0.803	-7.0
Brozzo et al. Criterion					
I	-0.253	-0.245	0.524	0.563	3.2
II	-0.321	-0.318	0.562	0.600	0.9
III	-0.420	-0.424	0.611	0.656	-1.0
IV	-0.589	-0.642	0.709	0.828	-9.0
Oh et al. Criterion					
I	-0.253	-0.245	0.524	0.563	3.2
II	-0.321	-0.318	0.562	0.600	0.9
III	-0.420	-0.424	0.611	0.656	-1.0
IV	-0.589	-0.648	0.721	0.841	-10.0
Oyane et al. Criterion					
I	-0.253	-0.253	0.575	0.617	0.0
II	-0.321	-0.324	0.593	0.633	-0.9
III	-0.420	-0.424	0.611	0.656	-1.0
IV	-0.589	-0.589	0.602	0.721	0.0

For commercial purity aluminium, values of the material parameter '*C*' and the constant *A* used in the Oyane et al. [21] criterion are : *C*=0.7234 and *A* = 0.5737. Least square fitting of the failure strain data in equation (28) yields the constant *B* and the material parameter '*C*' in Murty et al. criterion [23,24]. These values are : *B* = -0.9084 and *C* = 0.404.

Specifying the axial strain (ε_z) and using the relation for the strain ratio parameter (β), the effective strain ($\bar{\varepsilon}$) is obtained from equation (17). Using the material parameter '*C*' and the Kernel function of the fracture criterion in equation (2), the fracture potential (*p*) corresponding to the specified ε_z is determined. The procedure is repeated by increasing the value of ε_z and identified the fracture strain (ε_{zf}) when $p \to 1$.

For the comparison of the experimental and theoretical failure strains, the relative error is defined by

$$\text{Relative error(\%)} = 100\,\left(1 - \frac{theoretical\ result}{\exp erimental\ result}\right) \tag{31}$$

Table-4 gives a good comparison of experimental and predicted axial failure strains for the commercial purity Aluminium. Figure-2 shows a good comparison of experimental failure strains with those obtained from Murty et al. fracture criterion for the commercial purity aluminium. All the six ductile fracture criteria have predicted the axial failure strains with reasonable accuracy for all strain paths. The Oyane et al. criterion and Murty et al. criterion have been very successful with a relative error of less than one percent for all the strain paths.

8. Development of a Ductile Fracture Criterion

Since the axial and circumferential strains at fracture provides a measure of workability, an attempt is made here to establish a ductile fracture criterion based on the integrals of stress functions from the observations of failure strains for the commercial purity aluminium.

Integrals of stress function from the measured strains at the equatorial free surface of upsetting cylindrical specimens indicate the possibility of expressing the axial and circumferential failure strains (ε_{zf} and $\varepsilon_{\theta f}$) in the form:

$$\varepsilon_{\theta f} = \frac{3}{2}\int_0^{\bar{\varepsilon}_f} \frac{(\sigma_\theta - \sigma_H)}{\bar{\sigma}} d\bar{\varepsilon} \tag{32}$$

$$\varepsilon_{zf} = \frac{3}{2}\int_0^{\bar{\varepsilon}_f} \frac{(2\sigma_H - \sigma_\theta)}{\bar{\sigma}} d\bar{\varepsilon} \tag{33}$$

If $\varepsilon_{\theta f}$ and ε_{zf} exhibit linear relationship as in the present case (see Figure-2) for the commercial purity aluminium, the kernel function of Murty et al. [23,24] will be more appropriate for prediction of the ductile fracture. If the data of $\varepsilon_{\theta f}$ and ε_{zf} cannot exhibit linear relationship, one can alternatively express as

$$\varepsilon_{\theta f} = F(\varepsilon_{zf}) \tag{34}$$

Using equations (32) and (33) in equation (34), it is possible to establish a ductile fracture criterion based on the integrals of stress functions as

$$\int_0^{\bar{\varepsilon}_f} \frac{\sigma_\theta}{\bar{\sigma}} d\bar{\varepsilon} = \int_0^{\bar{\varepsilon}_f} \frac{\sigma_H}{\bar{\sigma}} d\bar{\varepsilon} + \frac{2}{3} F\left(\int_0^{\bar{\varepsilon}_f} \left(3\frac{\sigma_H}{\bar{\sigma}} - \frac{3}{2}\frac{\sigma_\theta}{\bar{\sigma}} \right) d\bar{\varepsilon} \right) \tag{35}$$

The material parameters in equation (35) will be nothing but the constants in the function F, which are to be obtained from the experimental failure strains.

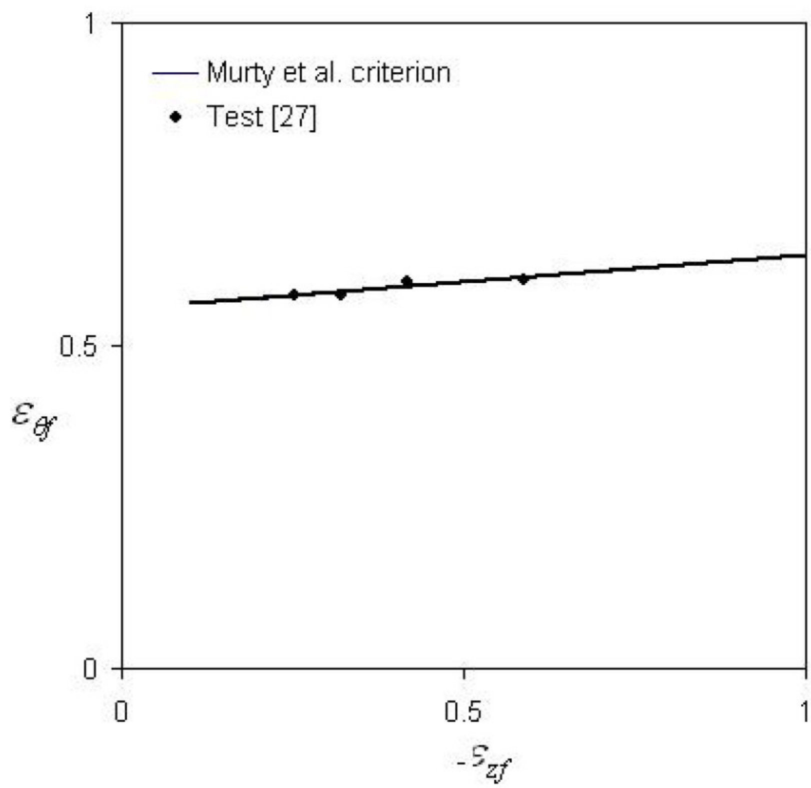

Figure 2. Forming Limit curve which relates the circumferential failure strain ($\varepsilon_{\theta f}$) in terms of axial failure strain (ε_{zf}) for the commercial purity aluminium.

CONCLUDING REMARKS

A comparative study has been made on six commonly used ductile fracture criteria to predict the occurrence of fracture in metal-forming processes, utilizing the test data from upset cylindrical specimens of commercial purity aluminium. A simple ductile fracture criterion in terms of integrals of stress functions, which takes into account the tensile stress ratio and triaxiality, is proposed. It is valid even for complex relationship between the axial and circumferential failure strains.

ACKNOWLEDGEMENTS

The authors wish to thank: VSSC Editorial Board for making necessary arrangements for reviewing this article prior to its clearance for publication; Dr. N.S. Babu (Systems Manager - Materials, Programme Planning and Evaluation Group) for his valuable comments / suggestions during the preparation of this article; Mr. G. Kothandaraman (General Manager, Mechanical Engineering Entity), Dr. P.P. Sinha (Deputy Director) for their encouragements; and Dr. K. Radhakrishnan, Director, VSSC for giving permission to publish this article.

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